KPK BOARD NOTES

GENERAL MATHEMATICS

10TH CLASS

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Unit 1 ALGEBRAIC FORMULAS AND APPLICATIONS

Algebraic Expressions

The expressions involving constants and variables connected by binary operations $(+, -, \times, \div)$ are called Algebraic expressions.

e.g $2x+3$, $12x$, $\frac{16}{x^2-1}$ etc

Rational expression:→ The expression of the form $\frac{P(x)}{q(x)}$, where $q(x) \neq 0$ and both $P(x), q(x)$ are algebraic expressions are called rational expression.

e.g $\frac{3x+2}{x^2-1}$, $\frac{2x^2+5}{3}$ etc.

Polynomial:- The algebraic expression of the form $P(x) = a_0 + a_1 x + \cdots + a_n x^n$ is called polynomial of degree n if $a_n \neq 0$ and all exponents are positive integers.

e.g $2x^2+5x+3$ is polynomial of degree 2.

$x^2 + x^{1/2} + 5$ is not a polynomial b/c exponent of 2nd term is $1/2$ which is not a positive integer.

(2)

Ex# 1.1

(1) Which of the expression behaves like a rational number in the following expressions:

Soln:- (i) $x+1$

It is a rational expression because

$x+1 = \frac{x+1}{1} = \frac{P(x)}{q(x)}$ form.

(ii) $\frac{x^2+1}{x-1}$

Soln:- It is clearly in $\frac{P(x)}{q(x)}$ form so it behaves like rational number.

(iii) $x^2+\frac{3}{x}$.

Soln:- It is not $\frac{P(x)}{q(x)}$ form so not rational.

(iv) $\frac{x^2+2x+3}{x^2+2}$

Soln:- It is in $\frac{P(x)}{q(x)}$ form so rational.

(v) $\frac{x^3+2x^2+3x}{x^2+4x+5}$

Soln:- It is rational as it is in $\frac{P(x)}{q(x)}$ form.

(2) Which expressions are polynomials.

Soln:- (i) $\sqrt{x+1}$ .. It is not a polynomial as the exponent is $1/2$

(ii) x^2+2x+3: It is a polynomial.

(iii) $x^2+\frac{1}{x}+1$: It is not a polynomial as in $\frac{1}{x}=x^{-1}$, the exponent is negative.

(iv) $8x^2y + 5xy$: It is a polynomial.

(v) $x^3 - \frac{1}{2x^2} + \frac{3}{4}$: It is not a polynomial because exponent of $\frac{1}{2x^2}$ is -2.

(3) Which of the following are rational expressions?

Solns:-

(i) $\frac{x^2+x+3}{x^3+1}$: It is rational expression

(ii) $\frac{x^3+4/x+2}{x^2+1}$: It is in $\frac{P(x)}{q(x)}$ form but $P(x) = x^3 + 4/x + 2$ is not polynomial so it is not rational.

(iii) $\sqrt{x^2+3} - 1$: It is not rational.

(iv) $\frac{1}{x}$: Rational as it is in $\frac{P(x)}{q(x)}$ form

(v) 2 : Rational as $2 = \frac{2}{1} = \frac{P(x)}{q(x)}$ form.

(vi) $\frac{3}{x^3-1}$: Rational as it is in $\frac{P(x)}{q(x)}$ form.

(4) Find the degrees of polynomials:

Soln:

(i) $x^3 + 2x^2 + 3x + 4$ degree = 3.

(ii) $x^2 + 2x + 3$ degree = 2

(iii) $\frac{1}{3}x + 4$ degree = 1

(iv) $5 = 5x^0$ degree = 0

(v) $-2x^2 + \frac{5}{3}x + 4$ degree = 2

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معزز ممبران: آپ کا وٹس ایپ گروپ ایڈ من "**اردو بکس**" آپ سے مخاطب ہے۔

آپ تمام ممبران سے گزارش ہے کہ:

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- گروپ میں معزز، پڑھے لکھے، سلجھے ہوئے ممبرز موجود ہیں اخلاقیات کی پابندی کریں اور گروپ رولز کو فالو کریں بصورت دیگر معزز ممبرز کی بہتری کی خاطر ریموو کر دیا جائے گا۔
- کوئی بھی ممبر کسی بھی ممبر کو انباکس میں میسیج، مس کال، کال نہیں کرے گا۔ رپورٹ پر فوری ریموو کر کے کاروائی عمل میں لائے جائے گی۔
- ہمارے کسی بھی گروپ میں سیاسی و فرقہ واریت کی بحث کی قطعاً کوئی گنجائش نہیں ہے۔
- اگر کسی کو بھی گروپ کے متعلق کسی قسم کی شکایت یا تجویز کی صورت میں ایڈ من سے رابطہ کیجئے۔
- سب سے اہم بات:

گروپ میں کسی بھی قادیانی، مرزائی، احمدی، گستاخِ رسول، گستاخِ امہات المؤمنین، گستاخِ صحابہ و خلفائے راشدین حضرت ابو بکر صدیق، حضرت عمر فاروق، حضرت عثمان غنی، حضرت علی المرتضیٰ، حضرت حسنین کریمین رضوان اللہ تعالیٰ اجمعین، گستاخ اہلبیت یا ایسے غیر مسلم جو اسلام اور پاکستان کے خلاف پراپیگنڈا میں مصروف ہیں یا ان کے روحانی و ذہنی سپورٹرز کے لئے کوئی گنجائش نہیں ہے لہذا ایسے اشخاص بالکل بھی گروپ جوائن کرنے کی زحمت نہ کریں۔ معلوم ہونے پر فوراً ریموو کر دیا جائے گا۔

- تمام کتب انٹرنیٹ سے تلاش / ڈاؤنلوڈ کر کے **فری آف کاسٹ** وٹس ایپ گروپ میں شیئر کی جاتی ہیں۔ جو کتاب نہیں ملتی اس کے لئے معذرت کر لی جاتی ہے۔ جس میں محنت بھی صَرف ہوتی ہے لیکن ہمیں آپ سے صرف دعاؤں کی درخواست ہے۔
- عمران سیریز کے شوقین کیلئے علیحدہ سے عمران سیریز گروپ موجود ہے۔
- **لیڈیز کے لئے الگ گروپ کی سہولت موجود ہے جس کے لئے ویریفکیشن ضروری ہے۔**
- اردو کتب / عمران سیریز یا سٹڈی گروپ میں ایڈ ہونے کے لئے ایڈ من سے وٹس ایپ پر بذریعہ میسیج رابطہ کریں اور جواب کا انتظار فرمائیں۔ برائے مہربانی اخلاقیات کا خیال رکھتے ہوئے موبائل پر کال یا ایم ایس کرنے کی کوشش ہر گز نہ کریں۔ ورنہ گروپس سے تو ریموو کیا ہی جائے گا بلاک بھی کیا جائے گا۔

نوٹ: ہمارے کسی گروپ کی کوئی فیس نہیں ہے۔ سب فی سبیل اللہ ہے

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اللہ تبارک تعالیٰ ہم سب کا حامی وناصر ہو

(4)

Reduction of rational expression into lowest form

Let $\frac{P(x)}{q(x)}$ be rational expression. Then if $P(x)$ and $q(x)$ can,t be factorized Then $\frac{P(x)}{q(x)}$ is said into lowest form.

Expl: $\frac{x^2+1}{x+1}$ is in simplest form. Here $P(x)=x^2+1$ and $q(x)=x+1$ can,t be factorized.

Expl 1. Reduce $\frac{x^2-1}{x+1}$ into Lowest form

Soln: $\frac{x^2-1}{x+1} = \frac{(x)^2-(1)^2}{x+1} = \frac{(x+1)(x-1)}{(x+1)} = x-1$

Expl 2: Reduce $\frac{x^2-5x+6}{x^2-9}$ into Lowest form

Soln: To convert it into lowest form we will factorize $P(x)=x^2-5x+6$ and $q(x)=x^2-9$

Now; $p(x)=x^2-5x+6$
$= x^2-2x-3x+6$
$= x(x-2)-3(x-2)$
$= (x-2)(x-3)$

$q(x) = x^2-9$
$=(x)^2-(3)^2$
$=(x+3)(x-3)$

Therefore, $\frac{x^2-5x+6}{x^2-9} = \frac{(x-2)(x-3)}{(x+3)(x-3)} = \frac{x-2}{x+3}$

Ex # 1.2

1. Which of The following expressions are in lowest form

(i) $\frac{x+1}{2x-1}$ (ii) $\frac{3x^2+3}{9x+27}$ (iii) $\frac{x^2+3x+2}{x+2}$ (iv) $\frac{x^2-4}{x+2}$

(5)

(v) $\frac{4x+8}{2}$ (vi) $\frac{4x^2+3}{x+3}$

Soln:- The expressions (i) and (vi) are in lowest form while (ii), (iii), (iv) and (v) are not in lowest

2. Reduce the following rational algebraic expression in their lowest form.

(i) $\frac{x^2-x-6}{x+2}$

Soln:- $P(x) = x^2-x+6$

$\Rightarrow P(x) = x^2-3x+2x-6$

$= x(x-3)+2(x-3)$

$= (x+2)(x-3)$

So; $\frac{x^2-x-6}{x+2} = \frac{(x+2)(x-3)}{(x+2)}$

$= x-3$

(ii) $\frac{x^3+1}{x^2-1}$

Soln:-

$x^3+1 = (x)^3+(1)^3$

$= (x+1)(x^2-x+1)$

$\because a^3+b^3 = (a+b)(a^2-ab+b^2)$

$x^2-1 = (x)^2-(1)^2$

$= (x+1)(x-1)$

$\therefore \frac{x^3+1}{x^2-1} = \frac{(x+1)(x^2-x+1)}{(x+1)(x-1)}$

$= \frac{x^2-x+1}{x-1}$

(iii) $\frac{x^2y+xy}{xy^2-xy}$

Soln:-

$x^2y+xy = xy(x+1)$

$xy^2-xy = xy(y-1)$

$\therefore \frac{x^2y+xy}{xy^2-xy} = \frac{xy(x+1)}{xy(y-1)}$

$= \frac{x+1}{y-1}$

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(iv) $\frac{x^4+2x^2+1}{x^4-1}$

Soln:- x^4+2x^2+1

$= (x^2)^2+2(1)(x^2)+(1)^2$

$= (x^2+1)^2$

$= (x^2+1)(x^2+1)$

$\because x^4-1 = (x^2)^2-(1)^2$

$= (x^2+1)(x^2-1)$

$\therefore \frac{x^4+2x^2+1}{x^4-1} = \frac{(x^2+1)(x^2+1)}{(x^2+1)(x^2-1)}$

$= \frac{x^2+1}{x^2-1}$

(v) $\frac{x^4+4}{x^2-2x+2}$

Soln:-

$x^4+4 = (x^2)^2+(2)^2$

$= (x^2)^2+(2)^2+2(x^2)(2)-2(x^2)(2)$

$= (x^2+2)^2-4x^2$

$= (x^2+2)-(2x)^2$

$= (x^2+2x+2)(x^2-2x+2)$

$\therefore \frac{x^4+4}{x^2-2x+2} = \frac{(x^2+2x+2)(x^2-2x+2)}{(x^2-2x+2)} = x^2+2x+2$

(vi) $\frac{z^3-1}{z^2+z+1}$

Soln:- As;

$z^3-1 = (z)^3-(1)^3$

$= (z-1)(z^2+z+1)$

$\because a^3-b^3 = (a-b)(a^2+ab+b^2)$

$\therefore \frac{z^3-1}{z^2+z+1} = \frac{(z-1)(z^2+z+1)}{(z^2+z+1)}$

$= z-1$

which is the required Lowest form

(7)

Sum, Difference and Product of rational Expressions.

Expl 1: Find the sum of $\frac{x+1}{x-1}$ and $\frac{2x-1}{x-1}$

Soln: $\frac{x+1}{x-1}+\frac{2x-1}{x-1}$

$= \frac{(x+1)+(2x-1)}{x-1} = \frac{x+1+2x-1}{x-1} = \frac{3x}{x-1}$

Expl 2: Simplify the rational expression

$\frac{2}{x+1}+\frac{5}{x-1}+\frac{1}{x^2-1}$

Soln: $\frac{2}{x+1}+\frac{5}{x-1}+\frac{1}{x^2-1}$

$= \frac{2}{x+1}+\frac{5}{x-1}+\frac{1}{(x+1)(x-1)}$ $\because x^2-1=(x+1)(x-1)$

$= \frac{2(x-1)+5(x-1)+1}{(x+1)(x-1)}$ $\because$ Taking L.C.M

$= \frac{2x-2+5x-5+1}{(x+1)(x-1)} = \frac{7x-6}{x^2-1}$

Expl 3 Find the product of $\frac{x^2-y^2}{2y^2}$ and $\frac{3y}{x+y}$

Soln: $\frac{x^2-y^2}{2y^2}\times\frac{3y}{x+y}$

$= \frac{(x+y)(x-y)}{2y^2}\times\frac{3y}{x+y}$ $\because x^2-y^2 = (x+y)(x-y)$

$= \frac{3(x-y)}{2y}$ Ans.

(8)

Ex # 1.3

(1). Find the sum of the following rational expressions

(i) $\frac{3x-1}{2x-2}$, $\frac{2}{x-1}$

Soln: $\frac{3x-1}{2x-2} + \frac{2}{x-1} = \frac{3x-1}{2(x-1)} + \frac{2}{x-1}$

$= \frac{3x-1+2(2)}{2(x-1)}$

$= \frac{3x-1+4}{2(x-1)} = \frac{3x+3}{2x-2}$

(ii) $\frac{2x+y}{xy}$, $\frac{3y}{x}$

Soln:- $\frac{2x+y}{xy} + \frac{3y}{x} = \frac{2x+y+y(3y)}{xy}$ by Taking the L.C.M.

$= \frac{2x+y+3y^2}{xy}$

(iii) $\frac{1}{y-1}$, $\frac{1}{y+1}$

Soln: $\frac{1}{y-1} + \frac{1}{y+1} = \frac{y+1+y-1}{(y-1)(y+1)} = \frac{2y}{(y)^2-(1)^2} = \frac{2y}{y^2-1}$

(iv) $\frac{2y}{y^2+1}$, $\frac{4y}{y^2-1}$

Soln: $\frac{2y}{y^2+1} + \frac{4y}{y^2-1} = \frac{2y(y^2-1)+4y(y^2+1)}{(y^2+1)(y^2-1)}$

$= \frac{2y^3-2y+4y^3+4y}{(y^2)^2-(1)^2}$

$= \frac{6y^3+2y}{y^4-1}$

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(v) $\frac{xy^2+y}{x+3}$, $\frac{x^2y+x}{x+3}$

Soln:- $\frac{xy^2+y}{x+3}+\frac{x^2y+x}{x+3}=\frac{xy^2+y+x^2y+x}{x+3}$

$=\frac{xy^2+x^2y+x+y}{x+3}=\frac{xy(x+y)+1(x+y)}{xy}$

$=\frac{(x+y)(xy+1)}{xy}$

2. Subtract 1st expression from 2nd.

(i) $\frac{15}{4y-3}$, $\frac{16}{4y+3}$

Soln:- $\frac{16}{4y+3}-\frac{15}{4y-3}$

$=\frac{16(4y-3)-15(4y+3)}{(4y+3)(4y-3)}=\frac{64y-48-60y-45}{(4y)^2-(3)^2}$

$=\frac{64y-60y-48-45}{16y^2-9}=\frac{4y-93}{16y^2-9}$

(ii) $\frac{1}{a+b}$, $\frac{1}{a-b}$

Soln:- $\frac{1}{a-b}-\frac{1}{a+b}=\frac{a+b-(a-b)}{(a-b)(a+b)}$

$=\frac{a+b-a+b}{(a)^2-(b)^2}=\frac{2b}{a^2-b^2}$

(iii) $\frac{x+y}{y}$, $\frac{x^2-y^2}{xy}$

Soln: $\frac{x^2-y^2}{xy}-\frac{x+y}{y}=\frac{x^2-y^2-x(x+y)}{xy}$

$=\frac{x^2-y^2-x^2-xy}{xy}=\frac{-y(x+y)}{xy}=-\frac{(x+y)}{x}$

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(iv) $\frac{2x+1}{x+1}$, $\frac{2x-1}{x-1}$

Soln: $\frac{2x-1}{x-1} - \frac{2x+1}{x+1}$

$= \frac{(2x-1)(x+1)-(2x+1)(x-1)}{(x-1)(x+1)}$

$= \frac{2x(x+1)-1(x+1)-x(2x+1)+1(2x+1)}{(x)^2-(1)^2}$

$= \frac{2x^2+2x-x-1-2x^2-x+2x+1}{x^2-1}$

$= \frac{4x-2x}{x^2-1} = \frac{2x}{x^2-1}$ Ans

3). Find the product of the following expressions.

(i) $\frac{49x^2-4y^2}{x^2-9}$, $\frac{x-3}{7x-2y}$

Soln: $\frac{49x^2-4y^2}{x^2-9} \times \frac{x-3}{7x-2y}$

$= \frac{(7x)^2-(2y)^2}{(x)^2-(3)^2} \times \frac{x-3}{7x-2y}$

$= \frac{(7x+2y)(7x-2y)}{(x+3)(x-3)} \times \frac{(x-3)}{(7x-2y)}$ by using $a^2-b^2=(a+b)(a-b)$

$= \frac{7x+2y}{x+3}$

(ii) $\frac{x^3-y^3}{x^2+2xy+y^2}$, $\frac{x+y}{x^2+xy+y^2}$

Soln: $\frac{x^3-y^3}{x^2+2xy+y^2} \times \frac{x+y}{x^2+xy+y^2}$

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$$= \frac{(x-y)(x^2+xy+y^2)}{(x+y)x} \times \frac{(x+y)}{(x^2+xy+y^2)} \qquad \because a^3-b^3 = (a-b)(a^2+ab+b^2)$$

$$= \frac{x-y}{x+y} \quad \text{Ans.}$$

(iii) $\frac{4x^2+12xy+9y^2}{2x^2+5xy+3y^2}, \quad \frac{3x+3}{x^2-1}$

Soln:- $\frac{4x^2+12xy+9y^2}{2x^2+5xy+3y^2} \times \frac{3x+3}{x^2-1}$

$$= \frac{4x^2+6xy+6xy+9y^2}{2x^2+3xy+2xy+3y^2} \times \frac{3(x+1)}{(x)^2-(1)^2}$$

$$= \frac{2x(2x+3y)+3y(2x+3y)}{x(2x+3y)+y(2x+3y)} \times \frac{3(x+1)}{(x+1)(x-1)}$$

$$= \frac{(2x+3y)(2x+3y)}{(x+y)(2x+3y)} \times \frac{3(x+1)}{(x+1)(x-1)}$$

$$= \frac{3(2x+3y)}{(x+y)(x-1)} \quad \text{Ans}$$

Note :- Instead of $5xy$, $15xy$ is given. The ans in the book is also wrong.

(iv) $\frac{x^3+a^3}{x+a}, \quad \frac{x+a}{x^3-a^3}$

Soln:- $\frac{x^3+a^3}{x+a} \times \frac{x+a}{x^3-a^3} = \frac{x^3+a^3}{x^3-a^3}.$

4) Divide the following rational expressions

(i) $\frac{2x^2+13x+15}{4x^2-1}$ by $\frac{2x^2+11x+5}{4x^2-1}$

Soln: $\dfrac{2x^2+13x+15}{4x^2-1} \div \dfrac{2x^2+11x+5}{4x^2-1}$

$= \dfrac{2x^2+10x+3x+15}{4x^2-1} \div \dfrac{2x^2+10x+x+5}{4x^2-1}$

$= \dfrac{2x(x+5)+3(x+5)}{4x^2-1} \div \dfrac{2x(x+5)+1(x+5)}{4x^2-1}$

$= \dfrac{(x+5)(2x+3)}{4x^2-1} \div \dfrac{(x+5)(2x+1)}{4x^2-1}$

$= \dfrac{(x+5)(2x+3)}{(4x^2-1)} \times \dfrac{(4x^2-1)}{(x+5)(2x+1)} = \dfrac{2x+3}{2x+1}$

(ii) $a+\dfrac{2}{x^2}$ by $\dfrac{a+2}{x^2}$

Soln:- $\dfrac{a}{1}+\dfrac{2}{x^2} \div \dfrac{a+2}{x^2}$

$= \dfrac{ax^2+2}{x^2} \div \dfrac{a+2}{x^2} = \dfrac{ax^2+2}{x^2} \times \dfrac{x^2}{a+2}$

$= \dfrac{ax^2+2}{a+2}$

(iii) $\dfrac{4-x^2}{x^2-9}$ by $\dfrac{12+6x}{x^2-6x+9}$

Soln:- $\dfrac{4-x^2}{x^2-9} \div \dfrac{12+6x}{x^2-6x+9}$

$= \dfrac{(2)^2-(x)^2}{(x)^2-(3)^2} \div \dfrac{6(2+x)}{x^2-3x-3x+9}$

$= \dfrac{(2+x)(2-x)}{(x+3)(x-3)} \div \dfrac{6(2+x)}{x(x-3)-3(x-3)}$

$= \dfrac{(2+x)(2-x)}{(x+3)(x-3)} \times \dfrac{(x-3)(x-3)}{6(2+x)} = \dfrac{(2-x)(3-x)}{6(x+3)}$

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(iv) $\frac{x^8-1}{4x^2-4}$ by $\frac{x^4-1}{x-2}$

Soln:- $\frac{x^8-1}{4x^2-4} \div \frac{x^4-1}{x-2}$

$$= \frac{(x^4)^2-(1)^2}{4(x^2-1)} \times \frac{(x-2)}{x^4-1} = \frac{(x^4+1)(x^4-1)}{4(x^2-1)} \times \frac{x-2}{(x^4-1)}$$

$$= \frac{(x-2)(x^4+1)}{4(x^2-1)}$$

5) Simplify the following expressions.

(i) $\frac{1}{a+b} - \frac{1}{a-b} + \frac{2a}{a^2-b^2}$

Soln:. $\frac{1}{a+b} - \frac{1}{a-b} + \frac{2a}{a^2-b^2} = \frac{a-b-(a-b)+2a}{(a+b)(a-b)}$

$$= \frac{a-b-a+b+2a}{a^2-b^2} = \frac{2a}{a^2-b^2}$$

(ii) Soln:- $\frac{x^2-x-20}{x^2-25} \times \frac{x^2-x-2}{x^2+2x-8} \div \frac{x+1}{x^2+25}$

$$= \frac{x^2-5x+4x-20}{(x)^2-(5)^2} \times \frac{x^2-2x+x-2}{x^2+4x-2x-8} \div \frac{x+1}{x^2+25}$$

$$= \frac{x(x-5)+4(x-5)}{(x+5)(x-5)} \times \frac{x(x-2)+1(x-2)}{x(x+4)-2(x+4)} \div \frac{x+1}{x^2+25}$$

$$= \frac{(x-5)(x+4)}{(x+5)(x-5)} \times \frac{(x+1)(x-2)}{(x-2)(x+4)} \times \frac{x^2+25}{(x+1)}$$

$$= \frac{x^2+25}{x+5} \text{ Ans.}$$

(14)

(iii) $\frac{2x^2+9x+9}{x^2-9} \times \frac{4x^2+8x+3}{x-3} \div \frac{2x+3}{3x-9}$

Soln: $\frac{2x^2+9x+9}{x^2-9} \times \frac{4x^2+8x+3}{x-3} \div \frac{2x+3}{3(x-3)}$

$= \frac{2x^2+6x+3x+9}{(x)^2-(3)^2} \times \frac{4x^2+6x+2x+3}{x-3} \div \frac{2x+3}{3(x-3)}$

$= \frac{2x(x+3)+3(x+3)}{(x+3)(x-3)} \times \frac{2x(2x+3)+1(2x+3)}{x-3} \times \frac{3(x-3)}{2x+3}$

$= \frac{(x+3)(2x+3)}{(x+3)(x-3)} \times \frac{(2x+3)(2x+1)}{x-3} \times \frac{3(x-3)}{(2x+3)}$

$= \frac{3(2x+3)(2x+1)}{x-3}$ Ans

Ex # 1.4

(1) Find the value of $5x+3$ at $x=\frac{1}{3}$ and $x=-1$

Soln:- For $x=\frac{1}{3}$:

$5x+3 = 5(\frac{1}{3})+3 = \frac{5}{3}+\frac{3}{1} = \frac{5+9}{3} = \frac{14}{3}$

For $x=-1$

$5x+3 = 5(-1)+3 = -5+3 = -2$

2) Find the value of $x^3+y^3+z^3$ at $x=3, y=1$ and $z=4$

Soln:- $x^3+y^3+z^3 = (3)^3+(1)^3+(4)^3$

$= 27+1+64$

$= 92$

(15)

3. Find the value of $H = \frac{2ab}{a+b}$ at $a=4, b=2$

Soln:- $H = \frac{2ab}{a+b}$. Putting the values of a and b

$= \frac{2(4)(2)}{4+2} = \frac{16}{6} = \frac{8}{3}$

4. $f(x) = x^3 - 2x^2 + x - 1$. Find $f(-\frac{1}{2})$

Soln:- Putting the value $x = -\frac{1}{2}$ in $f(x)$

$f(-\frac{1}{2}) = (-\frac{1}{2})^3 - 2(-\frac{1}{2})^2 - \frac{1}{2} - 1$

$= -\frac{1}{8} - 2(\frac{1}{4}) - \frac{1}{2} - 1$

$= \frac{-1}{8} - \frac{2}{4} - \frac{1}{2} - \frac{1}{1} = \frac{-1-4-4-8}{8} = \frac{-17}{8}$

5. Find the value of expression:

$\frac{x^4}{2} + \frac{y^4}{3} - \frac{z^4}{4}$ at $x=2, y=3, z=-2$

Soln:- $\frac{x^4}{2} + \frac{y^4}{3} - \frac{z^4}{4} = \frac{(2)^4}{2} + \frac{(3)^4}{3} - \frac{(-2)^4}{4}$

$= \frac{16}{2} + \frac{81}{3} - \frac{16}{4} = 8 + 27 - 4 = 31$

6. Find the value of $x^3 + 2xyz + z^3$ at $x=2, y=4, z=3$.

Soln:- $x^3 + 2xyz + z^3 = (2)^3 + 2(2)(4)(3) + (3)^3$

$= 8 + 48 + 27 = 83$

7. Find the value of $\frac{xyz + 3xy^2}{x+y}$ at $x=1, y=1, z=1$

Soln:- $\frac{xyz + 3xy^2}{x+y} = \frac{1(1)(1)(1) + 3(1)(1)^2}{1+1} = \frac{1+3}{2}$

$= \frac{4}{2} = 2$

(16)

ALGEBRAIC FORMULAE

(i) $(a+b)^2 = a^2+b^2+2ab$

(ii) $(a-b)^2 = a^2+b^2-2ab$

(iii) $(a+b)^2+(a-b)^2 = 2(a^2+b^2)$

(iv) $(a+b)^2-(a-b)^2 = 4ab$

Exp1 Find the value of a^2+b^2, if $a+b=4$, $a-b=2$

Soln:- We know that $(a+b)^2+(a-b)^2 = 2(a^2+b^2)$ —(i)

Putting the values in Eq(i) we get

$2(a^2+b^2) = (4)^2+(2)^2 = 16+4 = 20$

$\Rightarrow \frac{2(a^2+b^2)}{2} = \frac{20}{2} \Rightarrow a^2+b^2 = 10$

Exp2 Find the value of ab when $a+b=6$ and $a-b=2$.

Soln:- As we know that;

$4ab = (a+b)^2-(a-b)^2$ —(1)

Putting $a+b=6$ and $a-b=2$ in Eq(1)

$4ab = (6)^2-(2)^2$

$4ab = 36-4$

$4ab = 32$

÷ing both sides by 4.

$\frac{4ab}{4} = \frac{32}{4} \Rightarrow ab = 8$

(17)

Ex # 1.5

Simplify with the help of formulae

(i) $(x+y)^2+(x-y)^2$

Soln:- $(x+y)^2+(x-y)^2$

$= x^2+y^2+2xy+x^2+y^2-2xy = 2x^2+2y^2$

(ii) $(2x^2+3y^2)^2+(2x^2-3y^2)^2$

Soln:- $(2x^2+3y^2)^2+(2x^2-3y^2)^2$

Using the formula $(a+b)^2+(a-b)^2=2(a^2+b^2)$ —(1)

Putting $a=2x^2$ and $b=3y^2$ in (1)

$(2x^2+3y^2)^2+(2x^2-3y^2)^2 = 2[(2x^2)^2+(3y^2)^2]$

$= 2(4x^4+9y^4)$

(iii) $(x+4y^2)^2-(x-4y^2)^2$

Soln:- As we know that

$(a+b)^2+(a-b)^2 = 4ab$ —(i)

Putting $a=x$, $b=4y^2$ in (i) we get

$(x+2y^2)^2-(x-4y^2)^2 = 4(x)(4y^2) = 16xy^2$

(iv) $(3x+9y^2)^2-(3x-9y^2)^2$

$\because (a+b)^2-(a-b)^2 = 4ab$ —(i)

Putting $a=3x$ and $b=9y^2$ in (i) we get

$(3x+9y^2)^2-(3x-9y^2)^2 = 4(3x)(9y^2)$

$= 108xy^2$

(v) $(4x^2+y)^2+(4x^2-y)^2$

Soln:- $\because (a+b)^2+(a-b)^2 = 2(a^2+b^2)$ —— ①

Putting $a=4x^2$ and $b=y$ in Eq ① we get

$(4x^2+y)^2+(4x^2-y)^2 = 2[(4x^2)^2-(y)^2]$

$= 2(16x^4-y^2)$

(vi) $(5x+2y)^2-(5x-2y)^2$

Soln:- As we know that

$(a+b)^2-(a-b)^2 = 4ab$ —— (1)

Putting $a=5x$ and $b=2y$ in (1).

$(5x+2y)^2-(5x-2y)^2 = 4(5x)(2y) = 40xy$

2. Find the values of a^2+b^2 when,

(i) $a+b=2, a-b=1$

Soln We know that

$2(a^2+b^2)=(a+b)^2+(a-b)^2$ —— (1)

Putting $a+b=2$ and $a-b=1$ in ① we get

$2(a^2+b^2)=(2)^2+(1)^2$

$2(a^2+b^2)=4+1$

$\Rightarrow a^2+b^2=\frac{5}{2}$ dividing by 2.

Ans

(ii) $a+b=8, a-b=2$

Soln:- As we know that

$2(a^2+b^2)=(a+b)^2+(a-b)^2$ —— (i)

Putting $a+b=8$ and $a-b=2$ in (i)

$2(a^2+b^2)=(8)^2+(2)^2$

$2(a^2+b^2)=64+4$

$2(a^2+b^2)=68$

$\Rightarrow \frac{2(a^2+b^2)}{2}=\frac{68}{2}34$

$a^2+b^2=34$

(19)

3. Find the value of ab when;

(i) $a+b=50$, $a-b=16$

Soln: we know that

$4ab=(a+b)^2-(a-b)^2$

Putting the values we get

$4ab=(50)^2-(16)^2$

$4ab=2500-256$

$4ab=2244$

÷ing b.s by 4

$\frac{4ab}{4}=\frac{2244}{4}$

$ab=561$

(ii) $a+b=60$, $a-b=16$

Soln:-

As we know that

$4ab=(a+b)^2-(a-b)^2$

Putting the values

$4ab=(60)^2-(20)^2$

$4ab=3600-400$

$4ab=3200$

÷ing b.s by 4

$\frac{4ab}{4}=\frac{3200}{4}$

$ab=800$

(iii) $a+b=40$, $a-b=12$

Soln:-

$\because 4ab=(a+b)^2-(a-b)^2$

Putting the values

$4ab=(40)^2-(12)^2$

$4ab=1600-144$

$4ab=1556$

$\frac{4ab}{4}=\frac{1556}{4}$

$ab=389$

(iv) $a+b=6$, $a-b=2$

Soln:-

$\because 4ab=(a+b)^2-(a-b)^2$

Putting the values

$4ab=(6)^2-(2)^2$

$4ab=36-4$

$4ab=32$

÷ing b.s by 4

$\frac{4ab}{4}=\frac{32}{4}$

$ab=8$

(20)

Formula 3 $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$= a^2+b^2+c^2+2(ab+bc+ca)$

Note:- The formula contains three types of terms i.e $a^2+b^2+c^2$, $ab+bc+ca$ and $a+b+c$. If any two of them are given the 3rd can be find easily.

Ex# 1.6

1. Find the value of $a^2+b^2+c^2$ when:

(i) $ab+bc+ca=2$, $a+b+c=3$.

Soln Squaring both sides of $a+b+c=3$. i.e

$(a+b+c)^2 = 3^2$

$\Rightarrow a^2+b^2+c^2+2ab+2bc+2ca = 9$

$\Rightarrow a^2+b^2+c^2+2(ab+bc+ca) = 9$

$\Rightarrow a^2+b^2+c^2+2(2)=9 \Rightarrow a^2+b^2+c^2+4=9$

$\Rightarrow a^2+b^2+c^2=9-4=5$

(ii) $ab+bc+ca=8$, $a+b+c=4$.

Soln:- Squaring both sides of $a+b+c=4$.

$(a+b+c)^2 = 4^2$

$\Rightarrow a^2+b^2+c^2+2(ab+bc+ca) = 16$

or $a^2+b^2+c^2+2(8)=16$ $\because ab+bc+ca=8$

$\Rightarrow a^2+b^2+c^2+16=16$

$\Rightarrow a^2+b^2+c^2=16-16$

$\Rightarrow a^2+b^2+c^2=0$

(21)

(iii) $ab+bc+ca=8$ —(i) $a+b+c=6$ —(ii)

Soln:- Squaring both sides of eq(ii) we have

$$(a+b+c)^2 = 6^2$$

$$\Rightarrow a^2+b^2+c^2+2(ab+bc+ca) = 36$$

Putting the value of $ab+bc+ca=8$ we get.

$$a^2+b^2+c^2+2(8)=36$$

$$\Rightarrow a^2+b^2+c^2+16=36$$

$$\Rightarrow a^2+b^2+c^2=36-16=20$$

2. Find the values $a+b+c$ When:

(i) $ab+bc+ca=3$; $a^2+b^2+c^2=6$

Soln:- As we know that

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$

$$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca) \text{ —(i)}$$

Putting the values of $ab+bc+ca=3$, $a^2+b^2+c^2=6$ in (i)

$$(a+b+c)^2 = 6+2(3) = 6+6=12$$

Taking square root on both sides we get

$$\sqrt{(a+b+c)^2} = \sqrt{12} = \sqrt{4\times3} = \pm2\sqrt{3}$$

$$\Rightarrow a+b+c = \pm2\sqrt{3}$$

(ii) $ab+bc+ca=5$, $a^2+b^2+c^2=2$

Soln:- $\because (a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

Putting $ab+bc+ca=5$ and $a^2+b^2+c^2=2$ we get

$$(a+b+c)^2 = 2+2(5) = 2+10=12$$

(22)

Taking a square root we get

$a+b+c=\sqrt{12} = \sqrt{4\times3} = \pm2\sqrt{3}$

(iii) $ab+bc+ca=7,\ a^2+b^2+c^2=4.$

Soln:- $\because (a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$ —(i)

Putting the given values in (1) we get

$(a+b+c)^2 = 4 + 2(7) = 4+14 = 18$

Taking square root on both sides

$\sqrt{(a+b+c)^2} = \sqrt{18} = \sqrt{9\times2} = \pm3\sqrt{2}$

$\Rightarrow a+b+c = \pm3\sqrt{2}$

3. Find the value of $ab+bc+ca$ when:

(i) $a+b+c=6$ —(i), $a^2+b^2+c^2=2$ —(ii)

Soln:. Thaking square on both sides of (1)

$(a+b+c)^2 = 6^2$

$\Rightarrow a^2+b^2+c^2+2ab+2bc+2ca=36$

$a^2+b^2+c^2+2(ab+bc+ca)=36$

$\Rightarrow 2+2(ab+bc+ca)=36 \quad \because a^2+b^2+c^2=2$

$\Rightarrow 2(ab+bc+ca)=36-2$

$\Rightarrow 2(ab+bc+ca)=34$

$\Rightarrow ab+bc+ca=\frac{34}{2}=17 \quad \div$ing by 2

(23)

(ii) $a+b+c=9$ —(i) $a^2+b^2+c^2=13$ —(ii)

Soln:- Squaring both sides of Eq (i) we have

$(a+b+c)^2 = 9^2$

$\Rightarrow a^2+b^2+c^2+2(ab+bc+ca) = 81$

Putting $a^2+b^2+c^2=13$ we get

$13+2(ab+bc+ca)=81$

$\Rightarrow 2(ab+bc+ca)=81-13=68$

÷ ing b.sides by 2

$ab+bc+ca=\frac{68}{2}=34$

So $ab+bc+ca=34$

(iii) $a+b+c=7$ —(i), $a^2+b^2+c^2=67$ —(ii)

Squaring both sides of (i)

$(a+b+c)^2 = 7^2$

$\Rightarrow a^2+b^2+c^2+2(ab+bc+ca)=49$

$\Rightarrow 67+2(ab+bc+ca)=49 \quad \because a^2+b^2+c^2=67$

$\Rightarrow 2(ab+bc+ca)=49-67$

$\Rightarrow 2(ab+bc+ca)=-18$

÷ing b.s by 2

$\frac{2}{2}(ab+bc+ca) = -\frac{18}{2}$

$\boxed{ab+bc+ca=-9}$ Ans

Formulas 4, 5, 6 and 7

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

OR

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

or $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(a+b)(a^2 - ab + b^2) = a^3 + b^3$ and

$(a-b)(a^2 + ab + b^2) = a^3 - b^3$.

Expl 1. Find the values of $a^3 - b^3$ when $a-b = 8$
$ab = 2$, $a+b = 4$

Soln:- Given that $a-b=8$

Taking cube on both sides we have

$(a-b)^3 = 8^3$

$\Rightarrow a^3 - b^3 - 3ab(a-b) = 512$

Putting the values $ab = 2$ and $a-b = 8$

$a^3 - b^3 - 3(2)(8) = 512$

$a^3 - b^3 - 48 = 512$

$\Rightarrow a^3 - b^3 = 512 + 48$

$a^3 - b^3 = 560$

(25)

Exp: Find the continued product of

$(a+b)(a-b)(a^2+ab+b^2)(a^2-ab+b^2)$

Soln:- $(a+b)(a-b)(a^2+ab+b^2)(a^2-ab+b^2)$

$= (a+b)(a^2-ab+b^2)(a-b)(a^2+ab+b^2)$

$= (a^3+b^3)(a^3-b^3)$

$= (a^3)^2-(b^3)^2 \quad \because (x+y)(x-y)=x^2-y^2$

$= a^6-b^6$

Exp! Find the continued product of:

$(2x+3y)(2x-3y)(4x^2+12xy+9y^2)(4x^2-12xy+9y^2)$

Soln:- $(2x+3y)(2x-3y)(4x^2+12xy+9y^2)(4x^2-12xy+9y^2)$

$= (2x+3y)(4x^2-12xy+9y^2)(2x-3y)(4x^2+12xy+9y^2)$

$= \left[(2x)^3+(3y)^3\right]\left[(2x)^3-(3y)^3\right]$

$= (8x^3+27y^3)(8x^3-27y^3)$

$= (8x^3)^2-(27y^3)^2 \quad$ using $(a+b)(a-b)=a^2-b^2$

$= 64x^6-729y^6$

Ex # 1.7

1. Expand with the help of formulas:

(i) $(x+y)^3$

Soln:- $(x+y)^3 = (x)^3+(y)^3+3xy(x+y)$

$= x^3+y^3+3x^2y+3xy^2$

(26)

(ii) $(2x-3y)^3$

Soln:- $(2x-3y)^3 = (2x)^3 - (3y)^3 - 3(2x)(3y)(2x-3y)$

$= 8x^3 - 27y^3 - 18xy(2x-3y)$

$= 8x^3 - 27y^3 - 36x^2y + 54xy^2$

$= 8x^3 - 36x^2y + 54xy^2 - 27y^3$

(iii) $\left(\frac{1}{x} + \frac{2}{3}y\right)^3$

Soln:- $\left(\frac{1}{x} + \frac{2}{3}y\right)^3 = \left(\frac{1}{x}\right)^3 + \left(\frac{2}{3}y\right)^3 + 3\left(\frac{1}{x}\right)\left(\frac{2}{3}y\right)\left(\frac{1}{x} + \frac{2}{3}y\right)$

$= \frac{1}{x^3} + \frac{8}{27}y^3 + 2\frac{y}{x}\left(\frac{1}{x} + \frac{2}{3}y\right)$

$= \frac{1}{x^3} + \frac{8}{27}y^3 + 2\frac{y}{x^2} + \frac{4}{3}\frac{y^2}{x}$

2. Reduce each of the following into $(x \pm y)^3$

(i) $x^3 + 6xy(x+2y) + 8y^3$

Soln:- $x^3 + 6xy(x+2y) + 8y^3$

$= (x)^3 + 3(x)(2y)(x+2y) + (2y)^3 = (x+2y)^3$

(ii) $x^3 - 6x^2y - 12xy^2 - 8y^3$

Soln:- $x^3 - 6x^2y - 12xy^2 - 8y^3$

$= x^3 - 6xy(x+2y) - 8y^3$

$= (x)^3 - 3(x)(2y)(x+2y) - (2y)^3$

$= (x-2y)^3$

3. Find the value of $a^3 + b^3$ when $a+b=6$, $ab=4$

Soln:- Taking cube of $a+b=6$

$(a+b)^3 = 6^3$ (27)

$\Rightarrow a^3+b^3+3ab(a+b) = 216$ —(i)

Putting $a+b=6$ and $ab=4$ in (i) we get

$a^3+b^3+3(4)(6) = 216$

$a^3+b^3+72 = 216$

$\Rightarrow a^3+b^3 = 216-72 = 144$

4) Evaluate x^3-y^3, when $x-y=2$ and $xy=12$

Soln:- Taking cube on b.s of $x-y=2$

$\therefore (x-y)^3 = 2^3$

$\Rightarrow x^3-y^3-3xy(x-y) = 8$ —(i)

Putting $x-y=2$ and $xy=12$ in (i)

$x^3-y^3-3(12)(2) = 8$

$x^3-y^3-72 = 8 \Rightarrow x^3-y^3 = 8+72 = 80$

5. Find the continued product.

(i) $(x+2y)(x-2y)(x^2+4y^2+2xy)(x^2+4y^2-2xy)$

Soln:- $(x+2y)(x-2y)(x^2+4y^2+2xy)(x^2+4y^2-2xy)$

$= (x+2y)(x^2+4y^2-2xy)(x-2y)(x^2+4y^2+2xy)$

$= \{(x)^3+(2y)^3\}\{(x)^3-(2y)^3\}$

$= (x^3+8y^3)(x^3-8y^3)$

$= (x^3)^2-(8y^3)^2 \quad \because (a+b)(a-b)=a^2-b^2$

$= x^6-64y^6$

(28)

(ii) Soln

$(2a+3b)(2a-3b)(4a^2+6ab+9b^2)(4a^2-6ab+9b^2)$

$= (2a+3b)(4a^2-6ab+9b^2)(2a-3b)(4a^2+6ab+9b^2)$

$= [(2a)^3+(3b)^3][(2a)^3-(3b)^3]$

$= (8a^3+27b^3)(8a^3-27b^3)$

$= (8a^3)^2-(27b^3)^2 = 64a^6-729b^6$

6. Find the values of a^3+b^3 when

(i) $a+b=3,\ a-b=5,\ ab=4$

Soln:- As $a^3+b^3=(a+b)(a^2+b^2-ab)$ —— (i)

Now we will find the value of a^2+b^2

$\because 2(a^2+b^2)=(a+b)^2+(a-b)^2 = 3^2+5^2$

$2(a^2+b^2)=9+25=34$

$\Rightarrow a^2+b^2=\frac{34}{2}=17$

Putting the values in Eq(i) we get

$a^3+b^3= 3[17-4] = 3(13) = 39$

(ii) $a+b=2,\ a-b=8,\ ab=16$

Soln:- As $a^3+b^3=(a+b)(a^2+b^2-ab)$ —— (i)

But, $2(a^2+b^2)=(a+b)^2+(a-b)^2 = 2^2+8^2$

$2(a^2+b^2)=4+64=68 \Rightarrow a^2+b^2=\frac{68}{2}=34$

Putting $a^2+b^2=34$ and $ab=16$ in (i)

$a^3+b^3= 2(34-16) = 2(18) = 36$

(29)

(iii) $a+b=1,\ a-b=4,\ ab=12.$

Soln- As $a^3+b^3=(a+b)(a^2+b^2-ab)$ ——(i)

But $2(a^2+b^2)=(a+b)^2+(a-b)^2$ ——(ii)

but $a+b=1$ and $a-b=4$ so (ii) becomes

$2(a^2+b^2)=1^2+4^2=1+16=17$

÷ing b.s by 2 we get

$a^2+b^2=17/2$

Putting $a^2+b^2=\frac{17}{2}$ and $ab=12$ in Eq(i)

$a^3+b^3=1\left(\frac{17}{2}-12\right)=1\frac{7-24}{2}=\frac{-7}{2}$

Note The ans of Q6 in the books are wrong. Thes ans can be obtaind by taking the cube of a+b directly. But the given data will not allow us.

7. Find the value of a^3-b^3 when

(i) $a+b=6,\ a-b=10,\ ab=3$

Soln- As we know that $a^3-b^3=(a-b)(a^2+b^2+ab)$ —(i)

But $2(a^2+b^2)=(a+b)^2+(a-b)^2$

$\Rightarrow 2(a^2+b^2)=6^2+10^2=36+100=136$

$\Rightarrow \frac{2(a^2+b^2)}{2}=\frac{136}{2}^{68}\Rightarrow a^2+b^2=68$

Putting $a^2+b^2=68$ and $ab=3$ in Eq(i)

$a^3-b^3=10(68+3)=10(71)=710$

(36)

(ii) $a+b=6$, $a-b=10$, $ab=3$

Soln:- $\because a^3-b^3=(a-b)(a^2+b^2+ab)$ — (i)

But $2(a^2+b^2)=(a+b)^2+(a-b)^2$

Putting $a+b=6$ and $a-b=10$

$2(a^2+b^2)=6^2+10^2=36+100=136$

$\Rightarrow a^2+b^2=\frac{136}{2}=66$

Putting these values in Eq (i) we get

$a^3-b^3=10(66+3)=10(69)=690$

(iii) $a+b=1$, $a-b=1$, $ab=1$

Soln $\because a^3-b^3=(a-b)(a^2+b^2+ab)$ — (i)

and $2(a^2+b^2)=(a+b)^2+(a-b)^2$

$2(a^2+b^2)=1^2+1^2=1+1=2$

$\Rightarrow a^2+b^2=\frac{2}{2}=1$

Putting $a^2+b^2=1$ and $ab=1$ in Eq (i) we get

$a^3-b^3=1(1+1)=1(2)=2$

Surds:- Any number of the form $\sqrt[n]{a}$ which can,t be expressed as a friction of two integer is called surd e.g $\sqrt{2}, \sqrt{3}, \sqrt{2}+\sqrt{3}$ etc.

Surds of Second order:- A surd of the type $\sqrt{a}\pm\sqrt{b}$ where a and b are +ve real numbers.

(31)

Conjugate Surds. If $a+\sqrt{b}$ is a surd then $a-\sqrt{b}$ is called its conjugate.

Exp 1 Evaluate: $\frac{1}{\sqrt{2}+1}$

Soln.- Multiplying numerator and denominator by $\sqrt{2}-1$ we have

$$\frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \cdot \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)}$$

$$= \frac{\sqrt{2}-1}{(\sqrt{2})^2-(1)^2} = \frac{\sqrt{2}-1}{2-1} = \frac{\sqrt{2}-1}{1}$$

Exp 2 Rationalize $\frac{3}{2\sqrt{3}+1}$

Soln.- $\frac{3}{2\sqrt{3}+1} = \frac{3}{2\sqrt{3}+1} \times \frac{2\sqrt{3}-1}{2\sqrt{3}-1}$

$$= \frac{3(2\sqrt{3}-1)}{(2\sqrt{3})^2-(1)^2} \quad \because a^2-b^2=(a+b)(a-b)$$

$$= \frac{3(2\sqrt{3}-1)}{12-1} \quad \because (2\sqrt{3})^2 = 2^2\cdot(\sqrt{3})^2 = 4\cdot 3 = 12$$

$$= \frac{3(2\sqrt{3}-1)}{11}$$

Exp Simplify $\frac{1}{\sqrt{5}-\sqrt{3}}$

Soln.- $\frac{1}{\sqrt{5}-\sqrt{3}} = \frac{1}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$= \frac{\sqrt{5}+\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{\sqrt{5}+\sqrt{3}}{5-3}$$

$$= \frac{\sqrt{5}+\sqrt{3}}{2}$$

(32)

Ex # 1.8

1. Which of the following are surds.

(i) 3 (ii) $\sqrt{4}$ (iii) $\sqrt{2}$ (iv) $3\sqrt{2}$ (v) $\sqrt{16}$

(vi) $\sqrt{\frac{9}{4}}$ (vii) $\frac{1}{\sqrt{5}}$ (viii) $2+\sqrt{3}$ (ix) $2-\sqrt{3}$

(x) $\sqrt{\frac{17}{4}}$

Soln :- The surds are: (iii), (iv), (vii), (viii), (ix) and x.

2. Separate 2nd order surds from the following.

(i) $\sqrt{3}$ (ii) $3\sqrt{3}$ (iii) $2^{1/2}$ (iv) $\sqrt[4]{5}$ (v) $\sqrt{7}$

Soln (i), (iii) and (v) are 2nd order surds.

Note: $3\sqrt{3} = 3^1 \cdot 3^{1/2} = 3^{1+1/2} = 3^{3/2}$ = Not of 2nd order.

3. Rationalize the following numbers.

(i) $\frac{1}{\sqrt{2}+3}$

Soln:- Multiplying by $\frac{\sqrt{2}-3}{\sqrt{2}-3}$

$$\frac{1}{\sqrt{2}+3} = \frac{1}{\sqrt{2}+3} \times \frac{\sqrt{2}-3}{\sqrt{2}-3}$$

$$= \frac{1(\sqrt{2}-3)}{(\sqrt{2})^2-(3)^2}$$

$$= \frac{\sqrt{2}-3}{2-9}$$

$$= \frac{\sqrt{2}-3}{-7} \text{ Ans}$$

(ii) $\frac{1}{\sqrt{3}-1}$

Soln :- Multiplying numerator and denominator by $\sqrt{3}+1$ we get

$$\frac{1}{\sqrt{3}-1} = \frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{\sqrt{3}+1}{(\sqrt{3})^2-(1)^2}$$

$$= \frac{\sqrt{3}-1}{3-1} = \frac{\sqrt{3}-1}{2}$$

(33)

(iii) $\frac{1}{2\sqrt{3}-1}$.

Soln:- Multiplying num and denumerator by $2\sqrt{3}+1$

$$\frac{1}{2\sqrt{3}-1} = \frac{1}{2\sqrt{3}-1} \cdot \frac{2\sqrt{3}+1}{2\sqrt{3}+1} = \frac{2\sqrt{3}+1}{(2\sqrt{3}-1)(2\sqrt{3}+1)}$$

$$= \frac{2\sqrt{3}+1}{(2\sqrt{3})^2-(1)^2} = \frac{2\sqrt{3}+1}{12-1}$$

$(2\sqrt{3})^2 = 2^2(\sqrt{3})^2 = 4\cdot3 = 12$

$$= \frac{2\sqrt{3}+1}{11}$$

(iv) $\frac{1}{\sqrt{5}-\sqrt{3}}$

Soln:- Multiplying numerator and denum by $\sqrt{5}+\sqrt{3}$

$$\frac{1}{\sqrt{5}-\sqrt{3}} = \frac{1}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{(\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})}$$

$$= \frac{\sqrt{5}+\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{\sqrt{5}+\sqrt{3}}{5-3} = \frac{\sqrt{5}+\sqrt{3}}{2}$$

(v) $\frac{1}{\sqrt{7}+\sqrt{3}}$.

Soln:- $$\frac{1}{\sqrt{7}+\sqrt{3}} = \frac{1}{\sqrt{7}+\sqrt{3}} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}$$

$$= \frac{\sqrt{7}-\sqrt{3}}{(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})}$$

$$= \frac{\sqrt{7}-\sqrt{3}}{(\sqrt{7})^2-(\sqrt{3})^2} = \frac{\sqrt{7}-\sqrt{3}}{7-3}$$

$$= \frac{\sqrt{7}-\sqrt{3}}{4}$$ Ans.

(34)

Ex#1.9

1. Encircle 'T' for true and 'F' for false.

(i) x^2+3x+5 is a polynomial (T)

(ii) $\frac{x}{x^2+1}$ is a rational expression (T)

(iii) $\frac{x-1}{x^2-1}$ is in Lowest form. (F)

(iv) $\frac{1}{2}$ is not rational expression. (F)

(v) In $\frac{p(x)}{q(x)}$, $q(x)$ must be zero. (F)

2. Choose the correct option.

(i) The degree of x^3-4x^2+3 is

(a) 2 (b) ✓3 (c) 1 d) 4

(ii) $(a+b)^2+(a-b)^2=$

a) $3(a^2+b^2)$ b) ✓$2(a^2+b^2)$ c) a^2+b^2 d) $4ab$

(iii) $(a+b)^2-(a-b)^2=$

a) ab b) ✓$4ab$ c) $2ab$ d) $-4ab$

(iv) $a^3+b^3+3a^2b+3ab^2=$

a) $(a-b)^3$ (b) ✓$(a+b)^3$ (c) a^3+b^3 d) a^3

3. Write a suitable number/word in the blanks

(i) The rationalized form of $\frac{1}{\sqrt{2}-1}$ = $\sqrt{2}+1$

(ii) The process in which surd appear in the numerator only is called rationalization

(iii) $(a-b)^3 = a^3-b^3-3ab(a-b)$

(iv) $(\sqrt{5})^3-(\sqrt{3})^3 = (\sqrt{5}-\sqrt{3})(8+\sqrt{15})$

$(a-b)(a^2+b^2+ab)$
$(\sqrt{5}-\sqrt{3})(5+3+\sqrt{3\cdot 5})$
$(\sqrt{5}-\sqrt{3})(8+\sqrt{15})$

Unit 2

FACTORIZATION

Def:- The process to express the algebraic expression in a product of expressions is called factorization.

Factorization of $Kx+Ky+Kz$:-

To factorized $Kx+Ky+Kz$, just take K common i.e $Kx+Ky+Kz = K(x+y+z)$

Exp1: Factorize the expression $2ax+4ay-8az$

Soln:- $2ax+4ay-8az = 2a(x+2y-4z)$

Exp2:- Factorize $8a^3b^2x + 8a^3b^2y$

Soln:- Taking $8a^3b^2$ common we get

$$8a^3b^2x + 8a^3b^2y = 8a^3b^2(x+y)$$

EX:2.1

Factorize the following expressions:

(1) $2x - 8y$

Soln:- Taking 2 common we get

$$2x-8y = 2(x-4y)$$

2) $3x^3+6x^2$

Soln:- Taking $3x^2$ as a common we get

$$3x^3+6x^2 = 3x^2(x+2).$$

(36)

3) $4x^3y^3 - x^2y^2$

Soln:- Taking x^2y^2 common we get

$4x^3y^3 - x^2y^2 = x^2y^2(4xy-1)$

4) $2x + 2y - 4z$

Soln:- Taking 2 as common we have

$2x + 2y - 4z = 2(x+y-2z)$

5. $16x^4 - 4x^3 + 2x^2$

Soln:- Taking $2x^2$ as a common we get

$16x^4 - 4x^3 + 2x^2 = 2x^2(8x^2 - 2x + 1)$

6. $28x^2y^4 - 35xy^3 + 42x^4y$

Soln:- Taking $7xy$ common we get

$28x^2y^4 - 35xy^3 + 42x^4y = 7xy(4xy^3 - 5y^2 + 6x^3)$

7. $7abc - 14a^2b^2c^2$

Soln:- $7abc - 14a^2b^2c^2 = 7abc(1 - 2abc)$

8. $16x + 32xy^2$

Soln:- $16x + 32xy^2 = 16x(1 + 2y^2)$

9. $4x^2 + 8a^2x^2 - 4a^3x$

Soln:- $4x^2 + 8a^2x^2 - 4a^3x = 4x(x + 2a^2x - a^3)$

10. $45x^{10}y^4 - 63x^6y^7 + 72x^3y^3$

Soln:- Taking $9x^3y^3$ common we get.

$45x^{10}y^4 - 63x^6y^7 + 72x^3y^3 = 9x^3y^3(5x^7y - 7x^3y^4 + 8)$

Factorization of type: $ax + ay + bx + by$.

Sometimes there is no common in all terms. So the given expression is rearranged and then partially common is taken out.

Exp1: Factorize $2ax^2 + 2bx^2 + a + 3bx + 3ax + b$

Soln: Arrange the common terms,

$$2ax^2 + 2bx^2 + 3bx + 3ax + b + a$$

$$= 2x^2(a+b) + 3x(a+b) + 1(a+b)$$

$$= (a+b)(2x^2 + 3x + 1)$$

Ex: 2.2

Factorize the following algebraic expressions.

1. $x^2 + xy + xz + yz$

Soln: $x^2 + xy + xz + yz$

$$= x(x+y) + z(x+y)$$

$$= (x+y)(x+z)$$

2. $a^2 + 2a + ab + 2b$

Soln: $a^2 + 2a + ab + 2b$

$$= a(a+2) + b(a+2)$$

$$= (a+2)(a+b)$$

3. $a^2 - 2b^3 - 2ab^2 + ab$

Soln: Arrange the common terms

(38)

$= a^2 + ab - 2b^3 - 2ab^2$

$= a(a+b) - 2b^2(b+a) = (a+b)(a-2b^2)$

4. $ax - a - x + 1$

Soln:- $ax - a - x + 1$

$= a(x-1) - 1(x-1) = (x-1)(a-1)$

5. $3x^3 - 2x^2 + 18x - 12$

Soln:- $3x^3 - 2x^2 + 18x - 12$

$= x^2(3x-2) + 6(3x-2) = (3x-2)(x^2+6)$

6. $3x + y + 3ax + ay + 3bx + by$

Soln:- $3x + y + 3ax + ay + 3bx + by$

$= 3x + 3ax + 3bx + y + ay + by$

$= 3x(1 + a + b) + y(1 + a + b) = (1+a+b)(3x+y)$

7. $2y^5 + 3y^4 + 2y^3 + 3y^2 + 6y + 9$

Soln:- Arrange the common terms

$2y^5 + 2y^3 + 6y + 3y^4 + 3y^2 + 9$

$= 2y(y^4 + y^2 + 3) + 3(y^4 + y^2 + 3) = (y^4 + y^2 + 3)(2y + 3)$

8. $x^2 - xy + xz + 3x - 3y - yz$

Soln:- Arrange the common factors

$x^2 - xy + 3x - 3y + xz - yz$

$= x(x-y) + 3(x-y) + z(x-y)$

$= (x-y)(x+3+z)$

Factorization of the type $a^2 \pm 2ab + b^2$

If The given algebraic expression can be easily convertable in $a^2 \pm 2ab + b^2$ form The the factorization will be:

$$a^2 \pm 2ab + b^2 = (a \pm b)^2 = (a \pm b)(a \pm b)$$

Expl 1: Factorize $4x^2 + 12xy + 9y^2$

Soln:- $4x^2 + 12xy + 9y^2$

$= (2x)^2 + 2(2x)(3y) + (3y)^2$

$= (2x+3y)^2 = (2x+3y)(2x+3y)$

Expl 2:- Factorize $5x^3 - 30x^2 + 45x$

Soln:- $5x^3 - 30x^2 + 45x = 5x(x^2 - 6x + 9)$

$= 5x[(x)^2 - 2(x)(3) + (3)^2]$

$= 5x(x-3)^2 = 5x(x-3)(x-3)$

Ex 2.3

Factorize the following expressions

1. $x^2 + 4x + 4$

Soln:- $x^2 + 4x + 4 = (x)^2 + 2(x)(2) + (2)^2 = (x+2)^2$

$= (x+2)(x+2)$

2. $y^2 - 8y + 16 = (y)^2 - 2(y)(4) + (4)^2$

$= (y-4)^2$

$= (y-4)(y-4)$

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3. $9a^2+12ab+4b^2$

Soln:: $9a^2+12ab+4b^2 = (3a)^2+2(3a)(2b)+(2b)^2$

$= (3a+2b)^2 = (3a+2b)(3a+2b)$

4. $-2x^2+12x-18$

Soln:: $-2x^2+12x-18 = -2(x^2-6x+9)$

$= -2[(x)^2-2(x)(3)+(3)^2]$

$= -2(x-3)^2 = -2(x-3)(x-3)$

5. $25(a+b)^2-20(a+b)c+4c^2$ —(i)

Soln:: Let $x = a+b$. Then putting x in Eq(i)

$25(a+b)^2-20(a+b)c+4c^2 = 25x^2-20xc+4c^2$

$= (5x)^2-2(5x)(2c)+(2c)^2$

$= (5x-2c)^2 = (5x-2c)(5x-2c)$

Putting again the value x we get

$= [5(a+b)-2c][5(a+b)-2c]$

$= (5a+5b-2c)(5a+5b-2c)$

6. $49y^2-14yz+z^2$

Soln:- $49y^2-14yz+z^2 = (7y)^2-2(7y)(z)+(z)^2$

$= (7y-z)^2 = (7y-z)(7y-z)$

7. $16a^2+9b^2-24ab$

Soln:- $16a^2+9b^2-24ab$

$= (4a)^2+(3b)^2-2(4a)(3b)$

$= (4a-3b)^2 = (4a-3b)(4a-3b)$

(41)

8. $4x^2+4+\frac{1}{x^2}$

Soln:- $4x^2+4+\frac{1}{x^2} = (2x)^2+2(2x)(1/x)+(1/x)^2$

$= (2x+1/x)^2 = (2x+1/x)(2x+1/x)$

Factorization of The type a^2-b^2

Working rule:- If The algebraic expression can converted easily into a^2-b^2 form Then the required factorization will be $a^2-b^2=(a+b)(a-b)$.

Expl:- Factorize a^4-b^4

Soln:- $a^4-b^4=(a^2)^2-(b^2)^2=(a^2+b^2)(a^2-b^2)$

$=(a^2+b^2)(a+b)(a-b)$

Expl:- Factorize $x^2y^2-x^2-y^2+1$

Soln:- $x^2y^2-x^2-y^2+1$

$= x^2(y^2-1)-1(y^2+1)$

$= (x^2-1)(y^2-1) = [(x)^2-(1)^2][(y)^2-(1)^2]$

$= (x+1)(x-1)(y+1)(y-1)$

Ex: 2.4

1. $16x^2-9$

Soln:- $16x^2-9 = (4x)^2-(3)^2=(4x+3)(4x-3)$

2. x^3y-xy^3

Soln:- $x^3y-xy^3= xy(x^2-y^2)= xy[(x)^2-(y)^2]$

$= xy(x+y)(x-y)$

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3. $2x^2-288$

Soln:- $2x^2-288 = 2(x^2-144)$

$= 2[(x)^2-(12)^2]$

$= 2(x+12)(x-12)$

4. $2(x-y)a^2-18(x-y)$

Soln:- $2(x-y)a^2-18(x-y)$

$= 2(x-y)[a^2-9]$ ∵ Taking $2(x-y)$ Common

$= 2(x-y)[(a)^2-(3)^2] = 2(x-y)(a+3)(a-3)$

5. $\frac{x^2}{y^2}-\frac{y^2}{x^2}$

Soln:- $\frac{x^2}{y^2}-\frac{y^2}{x^2} = \left(\frac{x}{y}\right)^2-\left(\frac{y}{x}\right)^2 = \left(\frac{x}{y}+\frac{y}{x}\right)\left(\frac{x}{y}-\frac{y}{x}\right)$

6. $\frac{81}{16}x^4-625$

Soln:- $\frac{81}{16}x^4-625 = \left(\frac{9}{4}x^2\right)^2-(25)^2$

$= \left(\frac{9}{4}x^2+25\right)\left(\frac{9}{4}x^2-25\right)$ ∵ $a^2-b^2=(a+b)(a-b)$

$= \left(\frac{9}{4}x^2+25\right)\left[\left(\frac{3}{2}x\right)^2-(5)^2\right]$

$= \left(\frac{9}{4}x^2+25\right)\left(\frac{3}{2}x+5\right)\left(\frac{3}{2}x-5\right)$

7. $16-25x^2$

Soln:- $16-25x^2 = (4)^2-(5x)^2 = (4+5x)(4-5x)$

8. ab^3+2b^2-ab-2

Soln:- ab^3+2b^2-ab-2

$= b^2(ab+2)-1(ab+1)$

$= (ab+2)(b^2-1) = (ab+2)[b^2-(1)^2] = (ab+2)(b+1)(b-1)$

9. $9y^2 - 64x^2$

Soln:- $9y^2 - 64x^2 = (3y)^2 - (8x)^2$

$= (3y + 8x)(3y - 8x)$

10. $x^8 - y^8$

Soln :- $x^8 - y^8 = (x^4)^2 - (y^4)^2$

$= (x^4 + y^4)(x^4 - y^4)$ $\because a^2 - b^2 = (a+b)(a-b)$

$= (x^4 + y^4)[(x^2)^2 - (y^2)^2]$

$= (x^4 + y^4)(x^2 + y^2)(x^2 - y^2)$ using $a^2 - b^2 = (a+b)(a-b)$

$= (x^4 + y^4)(x^2 + y^2)(x+y)(x-y)$

Factorization of the type $(a^2 \pm 2ab + b^2) - c^2$

Working rule:- This type of expression is converted into $a^2 - b^2$ form and will factorized easily.

Exp 1 Factorize $4a^2 + 4ab + b^2 - c^2$

Soln:- $4a^2 + 4ab + b^2 - c^2$

$= (2a)^2 + 2(2a)(b) + (b)^2 - c^2$

$= (2a + b)^2 - (c)^2$

$= (2a + b + c)(2a + b - c)$ $\because a^2 - b^2 = (a+b)(a-b)$

Exp 2: Factorize : $a^2 - b^2 - 2b - 1$

Soln:- $a^2 - b^2 - 2b - 1 = a^2 - (b^2 + 2b + 1)$

$= (a)^2 - (b+1)^2 = (a+b+1)[a - (b+1)]$

$= (a+b+1)(a-b-1)$

(44)

Exp3 Factorize $a^2+b^2+2ab-c^2-d^2-2cd$

Soln:- $a^2+b^2+2ab-c^2-d^2-2cd$

$= (a^2+b^2+2ab)-(c^2+d^2+2cd)$

$= (a+b)^2-(c+d)^2$

$= (a+b+c+d)[a+b-(c+d)]$

$= (a+b+c+d)(a+b-c-d)$

Ex: 2.5

Factorize the following expression

1. $x^2-2xy+y^2-4$

Soln:- $x^2-2xy+y^2-4$

$= [(x)^2-2(x)(y)+(y)^2]-(2)^2$

$= (x-y)^2-(2)^2 = (x-y+2)(x-y-2)$ $\because a^2-b^2 = (a+b)(a-b)$

2. $9a^2b^2-12abc+4c^2-25d^2$

Soln:- $9a^2b^2-12abc+4c^2-25d^2$

$= [(3ab)^2-2(3ab)(c)+(2c)^2]-25d^2$

$= (3ab-2c)^2-(5d)^2$

$= (3ab-2c+5d)(3ab-2c-5d)$ $\because a^2-b^2 = (a+b)(a-b)$

3. $1-a^2+2ab-b^2$

Soln:- $1-a^2+2ab-b^2 = 1-(a^2-2ab+b^2)$

$= (1)^2-(a-b)^2 = (1+a-b)[1-(a-b)]$

$= (1+a-b)(1-a+b)$

(45)

4. $1+2xy-y^2-x^2$

Soln:- $1+2xy-y^2-x^2 = 1-(x^2+y^2-2xy)$

$= (1)^2-(x-y)^2$

$= (1+x-y)[1-(x-y)] = (1+x-y)(1-x+y)$

5. $a^2+2ab+b^2-c^2+2cd-d^2$

Soln:- $a^2+2ab+b^2-c^2+2cd-d^2$

$= (a^2+2ab+b^2)-(c^2-2cd+d^2)$

$= (a+b)^2-(c-d)^2$

$= (a+b+c-d)[a+b-(c-d)]$

$= (a+b+c-d)(a+b-c+d)$

6. $2(xy+pq)-x^2-y^2+p^2+q^2$

Soln:- $2(xy+pq)-x^2-y^2+p^2+q^2$

$= 2xy+2pq-x^2-y^2+p^2+q^2$

$= p^2+q^2+2pq-x^2-y^2+2xy$

$= p^2+q^2+2pq-(x^2+y^2-2xy)$

$= (p+q)^2-(x-y)^2$

$= (p+q+x-y)[p+q-(x-y)]$ $\because a^2-b^2=(a+b)(a-b)$

$= (p+q+x-y)(p+q-x+y)$

7. $-a^2+b^2-2b+1$

Soln:- $-a^2+b^2-2b+1 = b^2-2b+1-a^2$

$= (b-1)^2-(a)^2 = (b-1+a)(b-1-a)$

(46)

8. $25a^2b^2 - 20abc + 4c^2 - 16d^2$

Soln:- $25a^2b^2 - 20abc + 4c^2 - 16d^2$

$= \left[(5ab)^2 - 2(5ab)(2c) + (2c)^2\right] - 16d^2$

$= (5ab - 2c)^2 - (4d)^2$

$= (5ab - 2c + 4d)(5ab - 2c - 4d) \quad \because a^2 - b^2 = (a+b)(a-b)$

Factorization of the type $a^4 + a^2b^2 + b^4$ or $a^4 + b^4$.

Working rule:- This type of expression can be reduced to $a^2 - b^2$ form by using completing square method.

Expl 2:- Factorize : $a^4 + 4b^4$

Soln:- $a^4 + 4b^4 = (a^2)^2 + (2b^2)^2 + 2(a^2)(2b^2) - 4a^2b^2$

$= (a^2 + 2b^2)^2 - (2ab)^2$

$= (a^2 + 2b^2 + 2ab)(a^2 + 2b^2 - 2ab)$

Expl 3:- Factorize $x^4 - 7x^2 + 1$

Soln:- $x^4 - 7x^2 + 1 = x^4 + 1 - 7x^2$

$= x^4 + 1 + 2x^2 - 7x^2 - 2x^2$ adding and subtracting $2x^2$

$= (x^2)^2 + (1)^2 + 2(x^2)(1) - 9x^2$

$= (x^2 + 1)^2 - (3x)^2 = (x^2 + 1 + 3x)(x^2 + 1 - 3x)$

EX : 2.6

1. $x^4 + 4$

Soln:- $x^4 + 4 = (x^2)^2 + (2)^2$

To make it perfect square, adding and sub $4x^2$

$x^4 + 4 = (x^2)^2 + (2)^2 + 4x^2 - 4x^2 = (x^2 + 2)^2 - (2x)^2$

$= (x^2 + 2 + 2x)(x^2 + 2 - 2x)$

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2. $4x^4+81$

Soln:- $4x^4+81 = (2x^2)^2+(9)^2$

For completing square adding and subtracting $2(9)(2x^2)$

$$4x^4+81 = (2x^2)^2+(9)^2+2(2x^2)(9)-2(2x^2)(9)$$
$$= (2x^2+9)^2-36x^2$$
$$= (2x^2+9)^2-(6x)^2 = (2x^2+9+6x)(2x^2+9-6x)$$

3. $4p^4+q^4$

Soln:- $4p^4+q^4 = (2p^2)^2+(q^2)^2$

adding and subtracting $2(2p^2)(q^2)$

$$4p^4+q^4 = (2p^2)^2+(q^2)^2+2(2p^2)(q^2)-2(2p^2)(q^2)$$
$$= (2p^2+q^2)^2-4p^2q^2$$
$$= (2p^2+q^2)^2-(2pq)^2$$
$$= (2p^2+q^2+2pq)(2p^2+q^2-2pq)$$

by using $a^2-b^2=(a+b)(a-b)$

4. x^4+x^2+1

Soln:- $x^4+x^2+1 = (x^2)^2+(1)^2+2(x^2)(1)-x^2$

$$= (x^2+1)^2-(x)^2$$
$$= (x^2+1+x)(x^2+1-x)$$

5. x^8+x^4+1

Soln:- $x^8+x^4+1 = x^8+2x^4+1-x^4$ ∵ adding and Subtract x^4

$$= (x^4)^2+2(x^4)(1)+1^2-x^4$$
$$= (x^4+1)^2-(x^2)^2 = (x^4+1+x^2)(x^4+1-x^2)$$
$$= (x^4+1-x^2)[x^4+2x^2+1-x^2]$$
$$= (x^4+1-x^2)[(x^2+1)^2-x^2]$$
$$= (x^4+1-x^2)(x^2+1+x)(x^2+1-x)$$

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6. $x^4+\frac{1}{x^4}-7$

Soln:- $x^4+\frac{1}{x^4}-7 = x^4+\frac{1}{x^4}-9+2$ ∵ adding and sub 2

$= x^4+\frac{1}{x^4}+2-9 = (x^2)^2+\frac{1}{(x^2)^2}+2(x^2)(\frac{1}{x^2})-9$

$= (x^2+\frac{1}{x^2})^2-(3)^2$

$= (x^2+\frac{1}{x^2}+3)(x^2+\frac{1}{x^2}-3)$ by using $a^2-b^2=(a+b)(a-b)$

7. $16x^4+4x^2y^2+y^4$

Soln:- $16x^4+4x^2y^2+y^4 = (4x^2)^2+4x^2y^2+(y^2)^2$

For perfect square adding and subtracting $4x^2y^2$

∴ $16x^4+4x^2y^2+y^4 = (4x^2)^2+4x^2y^2+4x^2y^2+(y^2)^2-4x^2y^2$

$= (4x^2)^2+8x^2y^2+(y^2)^2-4x^2y^2$

$= (4x^2+y^2)^2-(2xy)^2$

$= (4x^2+y^2+2xy)(4x^2+y^2-2xy)$

8. $5a^4b+11a^3b^3+9ab^4$

Soln:- $4a^4b+11a^3b^3+9ab^4=$

$= ab(4a^4+11a^2b^2+9b^4)$

$= ab[(2a^2)^2+(3b^2)^2+11a^2b^2]$

$= ab[(2a^2)^2+(3b^2)^2+12a^2b^2-a^2b^2]$

$= ab[(2a^2)^2+(3b^2)^2+2(2a^2)(3b^2)-a^2b^2]$

$= ab[(2a^2+3b^2)^2-(ab)^2]$

$= ab(2a^2+3b^2+ab)(2a^2+3b^2-ab)$

∵ $a^2-b^2=(a+b)(a-b)$

Factorization of the expression x^2+px+q

Working rule:- (i) Form factors of q such that sum of the factors is p

(ii) Replace p by the sum of factors of q.

Exp: Factorize: y^2+5y+6

Soln:- y^2+5y+6

$6 = 2\times3$ and $2+3=5$

$= y^2+(2+3)y+6$

$= y^2+2y+3y+6$

$= y(y+2)+3(y+2) = (y+3)(y+2)$

Ex: 2.7

Factorize the following expressions

1: x^2+6x+8

As $8 = 2\times4$ and $2+4=6$. So replace 6 by 2+4

x^2+6x+8

$= x^2+(2+4)x+8$

$= x^2+2x+4x+8$

$= x(x+2)+4(x+2)$

$= (x+2)(x+4)$

2. x^2-2x-8

Soln $-8 = 2\times-4$

and $2+(-4) = -2$

$\therefore x^2-2x-8$

$= x^2-(-4+2)x-8$

$= x^2+4x-2x-8$

$= x(x+4)-2(x+4)$

$= (x-2)(x+4)$

3. $y^2+6y-55$

$-55 = 11\times-5$

$= y^2+(11-5)y-55$

$= y^2+11y-5y-55$

$= y(y+11)-5(y+11)$

$= (y+11)(y-5)$

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4. $y^3 - 8y^2 + 15y$

$= y(y^2 - 8y + 15)$

$15 = -3 \times -5$

and $-3 + (-5) = -8$

$\therefore y(y^2 - 8y + 15)$

$= y[y^2 - 3y - 5y + 15]$

$= y[y(y-3) - 5(y-3)]$

$= y(y-3)(y-5)$

5. $z^2 - z - 20$

As $-20 = (-5)(4)$

and $-5 + 4 = 1$

So;

$z^2 - z - 20$

$= z^2 - (-5+4)z - 20$

$= z^2 + 5z - 4z - 20$

$= z(z+5) - 4(z+5)$

$= (z-4)(z+5)$

6. $-2x^3 + 48x^2 - 46x$

$= -2x(x^2 - 24x + 23)$

$= -2x[x^2 - 23x - x + 23]$

$= -2x[x(x-23) - 1(x-23)]$

$= -2x(x-23)(x-1)$

7. $-3x^4 + 6x^3 + 144x^2$

Taking $-3x^2$ as common

$-3x^2(x^2 - 2x - 48)$

$-48 = 6 \times (-8)$

and $6 - 8 = -2$

$\therefore -3x^2(x^2 - 2x - 48)$

$= -3x^2[x^2 - 8x + 6x - 48]$

$= -3x^2[x(x-8) + 6(x-8)]$

$= -3x^2(x-8)(x+6)$

8. $x^2 + 10x + 21$

Soln:- Here $21 = 7 \times 3$

and $7 + 3 = 10$

So, Replace 10 by $3 + 7$

$\therefore x^2 + 10x + 21$

$= x^2 + 3x + 7x + 21$

$= x(x+3) + 7(x+3)$

$= (x+3)(x+7)$

Factorization of the expression of type ax^2+bx+c

Working rule: (i) Find the product ac

(ii) Factorize ac in such a way that sum of factors is b

(iii) Replace c by that factors.

Expl 1:- $2x^2-5x+3$ Factorize it.

Soln:- Here $2\times3=6$

and $2+3=5$. So, replace 5 by 2+3

$\therefore\ 2x^2-5x+3 = 2x^2-(3+2)x+3$

$= 2x^2-3x-2x+3$

$= x(2x-3)-1(2x-3)$

$= (2x-3)(x-1)$

EX: 2.8

Factorize the following expressions

1. $2x^2-7x+3$

Soln-

$= 2x^2-(6+1)x+3$

$= 2x^2-6x-x+3$

$= 2x(x-3)-1(x-3) = (2x-1)(x-3)$

$2\times3=6$
$6=6\times1$.
$6+1=7$

2. $4x^2+8x+3$

Soln:- $4x^2+8x+3$

$= 4x^2+(2+6)x+3$

$= 4x^2+2x+6x+3$

$= 2x(2x+1)+3(2x+1)$

$= (2x+1)(2x+3)$

$\because\ 3\times4=12$
$12=6\times2$
and $6+2=8$

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3. $2y^2-3y+1$

Soln:- $2y^2-3y+1 = 2y^2-(2+1)y+1$

$= 2y^2-2y-y+1$

$= 2y(y-1)-1(y-1) = (y-1)(2y-1)$

$(-2)(-1) = 2$ and $(-2)+(-1) = 3$

4. $6x^3-15x^2+9x$

Soln:- $6x^3-15x^2+9x$

$= 3x(2x^2-5x+3)$

$= 3x[2x^2-(2+3)x+3]$

$= 3x[2x^2-2x-3x+3] = 3x[2x(x-1)-3(x-1)]$

$= 3x(x-1)(2x-3)$

$(-2)(-3) = 6$ and $(-2)+(-3) = -5$

5. $2xy^2+8xy-24x$

Soln:- $2xy^2+8xy-24x$

$= 2x(y^2+4y-12)$

$= 2x[y^2+(6-2)y-12]$

$= 2x[y^2+6y-2y-12]$

$= 2x[y(y+6)-2(y+6)]$

$= 2x(y+6)(y-2)$

-12: 6×-2

6. $-12y^2+5y+2$

$= -12y^2+8y-3y+2$

$= -4y(3y-2)-1(3y-2)$

$= (3y-2)(-4y-1)$ Ans

-24: -3×8

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7. $2x^2+13x+20$

$2\times20 = 40$, 8×5

Soln:- $2x^2+13x+20$

$= 2x^2+(5+8)x+20 = 2x^2+5x+8x+20$

$= x(2x+5)+4(2x+5)$

$= (2x+5)(x+4)$

8. $4x^2+20x+21$

$21\times4 = 84$, 14×6 and $14+6=20$

Soln:- $4x^2+20x+21$

$= 4x^2+(14+6)x+21$

$= 4x^2+14x+6x+21$

$= 2x(2x+7)+3(2x+7) = (2x+7)(2x+3)$

9. $-6x^3y-20x^2y^2-6xy^3$

Soln:- $-6x^3y-20x^2y^2-6xy^3 = -2xy[3x^2+10xy+3y^2]$

$= -2xy[3x^2+(9+1)xy+3y^2]$

$= -2xy[3x^2+9xy+xy+3y^2]$

$= -2xy[3x(x+3y)+y(x+3y)]$

$= -2xy(x+3y)(3x+y)$

$3\times3=9$, 9×1, $9+1=10$

10. $8x^2+23x-3$

Soln:- $8x^2+23x-3$

$= 8x^2+(24-1)x-3$

$= 8x^2+24x-x-3$

$= 8x(x+3)-1(x+3)$

$= (x+3)(8x-1)$

$-3(8) = -24$, 24×-1 and $24-1=23$

(54)

Factorization of the type $a^3+3a^2b+3ab^2+b^3$ and $a^3-3a^2b+3ab^2-b^3$

As $a^3+3a^2b+3ab^2+b^3=(a+b)^3=(a+b)(a+b)(a+b)$

Similarly

$a^3-3a^2b+3ab^2-b^3=(a-b)^3=(a-b)(a-b)(a-b)$

Ex: 2.9

Factorize the following expressions

1. $x^3+3x^2y+3xy^2+y^3$

Soln: $x^3+3x^2y+3xy^2+y^3=(x)^3+3(x)^2y+3(x)(y)^2+(y)^3$

$=(x+y)^3=(x+y)(x+y)(x+y)$

2. $64x^3-144x^2y+108xy^2-27y^3$

Soln:- $64x^3-144x^2y+108xy^2-27y^3$

$=(4x)^3-3(4x)^2(3y)+3(4x)(3y)^2-(3y)^3$

$=(4x-3y)^3=(4x-3y)(4x-3y)(4x-3y)$

3. $125a^3+75a^2b+15ab^2+b^3$

Soln:- $125a^3+75a^2b+15ab^2+b^3$

$=(5a)^3+3(5a)^2(b)+3(5a)(b)^2+(b)^3$

$=(5a+b)^3=(5a+b)(5a+b)(5a+b)$

4. $27a^3+189a^2b+441ab^2+343b^3$

Soln:- $27a^3+189a^2b+441ab^2+343b^3$

(55)

$= (3a)^3 + 3(3a)^2(7b) + 3(3a)(7b)^2 + (7b)^3$

$= (3a+7b)^3 = (3a+7b)(3a+7b)(3a+7b)$

5. $\frac{a^3}{8} - \frac{1}{4}a^2b + \frac{1}{6}ab^2 - \frac{b^3}{27}$

Soln:- $\frac{a^3}{8} - \frac{1}{4}a^2b + \frac{1}{6}ab^2 - \frac{b^3}{27}$

$= \left(\frac{a}{2}\right)^3 - 3\left(\frac{a}{2}\right)^2\left(\frac{b}{3}\right) + 3\left(\frac{a}{2}\right)\left(\frac{b}{3}\right)^2 - \left(\frac{b}{3}\right)^3$

$= \left(\frac{a}{2} - \frac{b}{3}\right)^3 = \left(\frac{a}{2} - \frac{b}{3}\right)\left(\frac{a}{3} - \frac{b}{3}\right)\left(\frac{a}{3} - \frac{b}{3}\right)$

6. $x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$

Soln:- $x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$

$= (x)^3 + 3(x^2)\left(\frac{1}{x}\right) + 3(x)\left(\frac{1}{x}\right)^2 + \left(\frac{1}{x}\right)^3$

$= \left(x + \frac{1}{x}\right)^3 = \left(x + \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\left(x + \frac{1}{x}\right)$

7. $8a^3 + 48a^2b + 96ab^2 + 64b^3$

Soln:- $8a^3 + 48a^2b + 96ab^2 + 64b^3$

$= (2a)^3 + 3(2a)^2(4b) + 3(2a)(4b)^2 + (4b)^3$

$= (2a+4b)^3 = (2a+4b)(2a+4b)(2a+4b)$

8. $8x^3 - 4x + \frac{2}{3x} - \frac{1}{27x^3}$

Soln:- $8x^3 - 4x + \frac{2}{3x} - \frac{1}{27x^3}$

$= (2x)^3 - 3(2x)^2\left(\frac{1}{3x}\right) + 3(2x)\left(\frac{1}{3x}\right)^2 - \left(\frac{1}{3x}\right)^3$

$= \left(2x - \frac{1}{3x}\right)^3 = \left(2x - \frac{1}{3x}\right)\left(2x - \frac{1}{3x}\right)\left(2x - \frac{1}{3x}\right)$

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Factorization of the type $a^3 \pm b^3$

As we know that $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

Exp1 Factorize $x^3 - 64$

Soln:- $x^3 - 64 = (x)^3 + (4)^3 = (x+4)[(x)^2 + (x)(4) + 4^2]$

$= (x+4)(x^2+4x+16)$

Ex: 2.10

Factorize the following expressions.

1) $2x^3+16$

Soln:- $2x^3+16 = 2(x^3+8) = 2[(x)^3+(2)^3]$

$= 2(x+2)[(x)^2-(x)(2)+(2)^2]$

$= 2(x+2)(x^2-2x+4)$

2. a^3-64b^3

Soln:- $a^3-64b^3 = (a)^3-(4b)^3$

$= (a-4b)[(a)^2+(a)(4b)+(4b)^2]$

$= (a-4b)(a^2+4ab+16b^2)$

3. x^6-y^6

Soln:- $x^6-y^6 = (x^3)^2-(y^3)^2$

$= (x^3+y^3)(x^3-y^3) = [(x)^3+(y)^3][(x)^3-(y)^3]$

$= (x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$

4. $64y^3+27$

Soln:- $64y^3+27 = (4y)^3+(3)^3 = (4y+3)[(4y)^2-(4y)(3)+3^2]$

$= (4y+3)(16y^2-12y+9)$

5. $24x^3+3$

Soln:- $24x^3+3 = 3(8x^3+1) = 3[(2x)^3+(1)^3]$

$= 3(2x+1)[(2x)^2-(2x)(1)+1^2]$

$= 3(2x+1)(4x^2-2x+1)$

6. $a^3 + (c+d)^3$

Soln:- $a^3+(c+d)^3 = (a)^3 + (c+d)^3$

$= (a+c+d)\left[(a)^2 - a(c+d) + (c+d)^2\right]$

7. $x^{12} - y^{12}$

Soln:- $x^{12} - y^{12} = (x^6)^2 - (y^6)^2 = (x^6+y^6)(x^6-y^6)$

$= \left[(x^2)^3+(y^2)^3\right]\left[(x^3)^2-(y^3)^2\right]$

$= (x^2+y^2)(x^4-x^2y^2+y^4)(x^3+y^3)(x^3-y^3)$

$= (x^2+y^2)(x^4-x^2y^2+y^4)(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$

8. $343a^3 - 125b^3 + 7a - 5b$

Soln:- $343a^3 - 125b^3 + 7a - 5b$

$= (7a)^3 - (5b)^3 + 1(7a-5b)$

$= (7a-5b)(49a^2+35ab+25b^2) + 1(7a-5b)$

$= (7a-5b)[49a^2+35ab+25b^2+1]$

9. $1 - 27y^3$

Soln:- $1-27y^3 = (1)^3 - (3y)^3 = (1-3y)\left[(1)^2+(1)(3y)+(3y)^2\right]$

$= (1-3y)(1+3y+9y^2)$

10. $18(x-y)^3 - 144(a-b)^3$

Soln:- Let $x-y = A$ and $a-b = B$ Then;

$18(x-y)^3 - 144(a-b)^3 = 18A^3 - 144B^3 = 18\left[A^3 - 8B^3\right]$

$= 18\left[(A)^3-(2B)^3\right] = 18(A-2B)\left[(A)^2 + A(2B) + (2B)^2\right]$

$= 18(A-2B)(A^2+2AB+4B^2)$

Putting the values of A and B we get

$= 18(x-y-2(a-b))\left[(x-y)^2 + 2(x-y)(a-b) + 4(a-b)^2\right]$

$= 18(x-y-2a+2b)\left[(x-y)^2 + 2(x-y)(a-b) + 4(a-b)^2\right]$

Remainder theorem:- This theorem states that when a polynomial of degree $n \geq 1$ is divided by $x-r$ then Remainder $= P(r)$

Exp2 Using remainder theorem, find the remainder when x^3+4x^2+2x+3 is divided by $x+3$.

Sol:- Here $P(x) = x^3+4x^2+2x+3$

and $x-r = x+3 = x-(-3)$

$\Rightarrow r = -3$.

By remainder theorem;

$R = P(r) = P(-3) = (-3)^3+4(-3)^2+2(-3)+3$

$= -27+36-6+3 = 6$

$\Rightarrow$ Remainder $= 6$

Exp3: Find K, when $3x^3+9x^2-(3K-4)x+2$ is exactly divided by $x-1$

Soln Here $x-r = x-1 \Rightarrow r = 1$ and

$P(x) = 3x^3+9x^2-(3K-4)(x)+2$

By Remainder theorem;

$R = P(r) = P(1) = 3(1)^3+9(1)^2-(3K-4)(1)+2$

$= 3+9-3K+4+2$

$= 18-3K$

But given that it is exactly divisible so;

$R = 0 \Rightarrow 18-3K = 0$

$\Rightarrow 18 = 3K$

$\frac{3K}{3} = \frac{18}{3}$

$\boxed{K = 6}$

(59) Ex# 2.11

Find the remainder of the following polynomials by remainder Theorem

1. When $x^2-8x+11$ divided by $x-1$

Soln:- Here $P(x) = x^2-8x+11$ and $x-r = x-1 \Rightarrow r=1$

By remainder Theorem;

$$\text{Remainder} = P(r) = P(1) = (1)^2-8(1)+11$$
$$= 1-8+11 = 12-8 = 4$$

2. $2x^3+7x^2+4x+2$ divided by $x+3$

Soln:- Here $P(x) = 2x^3+7x^2+4x+2$ and $x-r = x-(-3)$ $\Rightarrow \boxed{r=-3}$

Now by remainder Theorem;

$$R = P(r) = P(-3) = 2(-3)^3+7(-3)^2+4(-3)+2$$
$$= 2(-27)+7(9)-12+2 = -54+63-12+2$$
$$R = -66+65 = -1$$

3. $3x^2+x-200$ divided by $x-4$

Soln:- Here $P(x) = 3x^2+x-200$ and $x-r = x-4$ $\Rightarrow \boxed{r=4}$

By remainder Theorem;

$$R = P(r) = P(4) = 3(4)^2+4-200$$
$$= 3(16)+4-200$$
$$= 48+4-200 = 52-200 = -148$$

4. Without performing the division, find the value of K when $2x^3-Kx^2-2Kx+2$ is exactly divisible by $x+1$

Soln Here $P(x) = 2x^3-Kx^2-2Kx+3x+2$, $x-r = x-(-1)$ $\Rightarrow \boxed{r=-1}$

By remainder Theorem;

$$R = P(-1) = 2(-1)^3-K(-1)^2-2K(-1)+3(-1)+2$$
$$= -2-K+2K-3+2$$
$$= K-3$$

But using the condition $\Rightarrow R = 0$

$\Rightarrow K - 3 = 0 \Rightarrow K = 3$

5. Find the value of 'a' when $x^3 - 4x^2 + x - 2 \div x - 1$

Sol:- Here $P(x) = x^3 - 4x^2 + a - 2$ and $x - r = x - 1$

$\Rightarrow \boxed{r = 1}$

According to the condition

Remainder $= 0$, but $R = P(r) = P(1)$

So $P(1) = 0$

$\Rightarrow (1)^3 - 4(1)^2 + a - 2 = 0$

$\Rightarrow 1 - 4 + a - 2 = 0 \Rightarrow a - 5 = 0 \Rightarrow \boxed{a = 5}$

Zero Of The polynomial:- Let $P(x)$ be a polynomial then those values of x for which $P(x) = 0$ are called zeros.

e.g $P(x) = x^2 - 7x + 12$, then $x = 3, 4$ are zeros.

Indeed; $P(3) = 3^2 - 7(3) + 12 = 9 - 21 + 12 = 0$

Factor Theorem:- If $P(x)$ is a polynomial of degree $n \geq 1$ is divided by $x - r$ such that $P(r) = 0$ then $x - r$ must be factor of $P(x)$ and r is root.

Factorization of the expression $ax^3 + bx^2 + cx + d$.

This type of expression is usually factorized by factor Theorem. Working rule is below.

(i) Find the zero of polynomial say $x = a$

(ii) Then $x - a$ will be factor of $P(x)$

(iii) Divide $P(x)$ by $x - a$.

(61)

Ex: 2.12

Factorize the following polynomials by factor Theorem:

1. x^3-2x^2-5x+6

Soln: Let $P(x)=x^3-2x^2-5x+6$

Putting $x=1$ i.e

$P(1)=1^3-2(1)^2-5(1)+6$

$=1-2-5+6$

$=7-7=0$

$\Rightarrow x=1$ is the root.

$\Rightarrow x-1$ is factor of $P(x)$

The remaining factors can be find by division

$$\begin{array}{r|l} & x^2-x-6 \\ x-1 & x^3-2x^2-5x+6 \\ & \underline{\pm x^3 \mp x^2} \\ & -x^2-5x \\ & \underline{\mp x^2 \pm x} \\ & -6x+6 \\ & \underline{\mp 6x \pm 6} \\ & x \end{array}$$

So the required factorization is

$x^3-2x^2-5x+6=(x-1)(x^2-x-6)$

$=(x-1)[x^2-3x+2x-6]$

$=(x-1)[x(x-3)+2(x-3)]$

$=(x-1)(x+2)(x-3)$

2. x^3+x^2-4x-4

Soln: Let $P(x)=x^3+x^2-4x+4$.

Putting $x=2$ we get;

$P(2)=2^3+2^2-4(2)+4$

$=8+4-8-4=0$

So $x=2$ is root of $P(x)$

or $x-2$ is factor of $P(x)$

for other factors division will be used

$$\begin{array}{r|l} & x^2+3x+2 \\ x-2 & x^3+x^2-4x+4 \\ & \underline{\pm x^3 \mp 2x^2} \\ & 3x^2-4x \\ & \underline{\pm 3x^2 \mp 6x} \\ & 2x+4 \\ & \underline{\pm 2x \mp 4} \end{array}$$

(62)

So, the required factorization is:

$$x^3+x^2-4x-4=(x-2)(x^2+3x+2)$$
$$=(x-2)[x^2+2x+x+2]$$
$$=(x-2)[x(x+2)+1(x+2)]$$
$$=(x-2)(x+1)(x+2)$$

3. $x^3-9x^2+23x-15$

Soln:- Let $P(x)=x^3-9x^2+23x-15$

Putting $x=1$ we get

$$P(1)=(1)^3-9(1)^2+23(1)-15$$
$$=1-9+23-15$$
$$=24-24=0$$

So $x=1$ is root of $P(x)$ and $x-1$ is factor.

For remaining factor divide $P(x)$ by $x-1$

$$\begin{array}{r|l} & x^2-8x+15 \\ x-1 & x^3-9x^2+23x-15 \\ & \underline{\pm x^3 \mp x^2} \\ & -8x^2+23x \\ & \underline{\mp 8x^2 \pm 8x} \\ & 15x-15 \\ & \underline{\pm 15x \mp 15} \end{array}$$

So the required factorization is

$$x^3-9x^2+23x-15=(x-1)(x^2-8x+15)$$
$$=(x-1)[x^2-5x-3x+15]$$
$$=(x-1)[x(x-5)-3(x-5)]$$
$$=(x-1)(x-3)(x-5) \text{ Ans}$$

4. $x^3-4x^2-3x+18$

Soln:- let $P(x)=x^3-4x^2-3x+18$

$$P(-2)=(-2)^3-4(-2)^2-3(-2)+18$$
$$=-8-16+6+18=0$$

So $x = -2$ is zero of $P(x)$ i.e $x+2$ is factor of $P(x)$

Now dividing $P(x)$ by $x+2$

$$\begin{array}{r|l} & x^2-6x+9 \\ \hline x+2 & x^3-4x^2-3x+18 \\ & \underline{\pm x^3 \pm 2x^2} \\ & -6x^2-3x \\ & \underline{\mp 6x^2 \mp 12x} \\ & 9x+18 \\ & \underline{\pm 9x \pm 18} \end{array}$$

So required factorization is

$$x^3-4x^2-3x+18 = (x+2)(x^2-6x+9)$$
$$= (x+2)[x^2-3x-3x+9]$$
$$= (x+2)[x(x-3)-3(x-3)]$$
$$= (x+2)(x-3)(x-3)$$

5. $x^3+2x^2-19x-20$

Soln: Let $P(x) = x^3+2x^2-19x-20$

$$P(-1) = (-1)^3+2(-1)^2-19(-1)-20 = -1+2+19-20$$
$$= 20-20 = 0$$

So $x = -1$ is root of $P(x)$ and $x+1$ is factor.

Now;

$$\begin{array}{r|l} & x^2+x-20 \\ \hline x+1 & x^3+2x^2-19x-20 \\ & \underline{\pm x^3 \pm x^2} \\ & x^2-19x \\ & \underline{\pm x^2 \pm x} \\ & -20x-20 \\ & \underline{\mp 20x \mp 20} \\ & 0 \end{array}$$

(64)

The required factorization is

$x^3+2x^2-19x-20=(x+1)(x^2+x-20)$

$=(x+1)[x^2+5x-4x-20]$

$=(x+1)[x(x+5)-4(x+5)]$

$=(x+1)(x+5)(x-4)$

6. $x^3-x^2-14x+24$

Soln let $P(x)=x^3-x^2-14x+24$

$\Rightarrow P(2)=2^3-2^2-14(2)+24$ putting $x=2$

$=8-4-28+24=32-32=0$

So $x=2$ is root of $P(x)$ and $x-2$ is its factor.

$$\begin{array}{r|l} & x^2+x-12 \\ \hline x-2 & x^3-x^2-14x+24 \\ & \underline{\mp x^3 \pm 2x^2} \\ & \quad x^2-14x \\ & \quad \underline{\mp x^2 \pm 2x} \\ & \quad\quad -12x+24 \\ & \quad\quad \underline{\pm 12x \mp 24} \\ & \quad\quad\quad 0 \end{array}$$

So The required factorization is:

$x^3-x^2-14x+24=(x-2)(x^2+x-12)$

$=(x-2)[x^2+4x-3x-12]$

$=(x-2)[x(x+4)-3(x+4)]$

$=(x-2)(x+4)(x-3).$

(65)

7. Prove that $x-1$ is a factor of x^3-7x+6. Also find the other factors.

Soln:- Here $P(x) = x^3-7x+6$

Putting $x = 1$;

$P(1) = 1^3 - 7(1) + 6$

$= 7 - 7 = 0$

$\Rightarrow x-1$ is factor of $P(x)$

Now for other factors we will divide $P(x)$ by $x-1$

$$
\begin{array}{r|l}
 & x^2 + x - 6 \\
x-1 & x^3 \quad -7x+6 \\
 & \underline{\pm x^3 \mp x^2} \\
 & x^2 - 7x \\
 & \underline{\pm x^2 \mp x} \\
 & -6x + 6 \\
 & \underline{\mp 6x \pm 6} \\
\end{array}
$$

The required factorization is

$x^3 - 7x + 6 = (x-1)(x^2 + x - 6)$

$= (x-1)[x^2 + 3x - 2x - 6]$

$= (x-1)[x(x+3) - 2(x+3)]$

$= (x-1)(x+3)(x-2)$

8. Prove that $x+2$ is the factor of x^3-6x^2+32.

Soln:- Let $P(x) = x^3 - 6x^2 + 32$

Putting $x = -2$

$P(-2) = (-2)^3 - 6(-2)^2 + 32$

$= -8 - 6(4) + 32 = 32 - 32 = 0$

$\Rightarrow x+2$ is factor of $P(x)$

Now;

$$
\begin{array}{r|l}
 & x^2 - 8x + 16 \\
x+2 & x^3 - 6x^2 \quad + 32 \\
 & \underline{x^3 + 2x^2} \\
 & -8x^2 \\
 & \underline{\mp 8x^2 \mp 16x} \\
 & 16x + 32 \\
 & \underline{-16x + 32} \\
 & 0 \\
\end{array}
$$

(66)

The required factorization is;

$$x^3-6x^2+32=(x+2)(x^2-8x+16)$$
$$=(x+2)[x^2-4x-4x+16]$$
$$=(x+2)[x(x-4)-4(x-4)]$$
$$=(x+2)(x-4)(x-4)$$

Ex: 2.13 (Objective type Questions)

1. Tick 'T' for true and 'F' for false.

(i) $x^2-4=(x+2)(x-2)$ (T)

(ii) $2x^2y+8xy=2xy(x+4y)$ (T)

(iii) $x^3+8=(x+2)(x^2-2x+4)$ (T)

(iv) $x^2-5x+6=(x+6)(x-1)$ (F)

(v) $x^4-y^4=(x-y)(x+y)(x^2+y^2)$ (T)

2. Fill in the blanks.

(i) $9x^2-16y^2=$ $(3x+4y)(3x-4y)$

(ii) $x^2-xy-xz+yz=$ $(x-y)(x-z)$

(iii) $(2a^2-3b^2)^2=$ $(2a^2-3b^2)(2a^2-3b^2)$

(iv) $x^3-27=$ $(x-3)(x^2+3x+9)$

(v) $x^2-11x+30=$ $(x-5)(x-6)$

3. Choose the correct option in the box.

(i) c (ii) a (iii) b (iv) b (v) c

(67)

Unit 3

ALGEBRAIC MANIPULATION

Highest Common factor (HCF):- The H.C.F of two or more polynomials is a polynomial of highest degree which divides exactly the given polynomials.

There are two methods for finding HCF
(i) By Factorization (ii) By division.

→ Working rule for finding HCF by Factorization
(i) Factorize the given polynomials
(ii) The common factors in all the polynomials is the required HCF

Expl Find the H.C.F of $ax^2+5ax+6a$, ax^3+9ax^2+14ax and $15ax^2-60a$

Soln:- $ax^2+5ax+6a = a(x^2+5x+6)$
$= a[x^2+2x+3x+6] = a[x(x+2)+3(x+2)]$
$= a(x+2)(x+3)$

$ax^3+9ax^2+14x = ax(x^2+9x+14)$
$= ax(x^2+7x+2x+14)$
$= ax[x(x+7)+2(x+7)]$
$= ax(x+7)(x+2)$

$15ax^2-60a = 15a(x^2-4) = 15a(x+2)(x-2)$

Common factors (C.F) = a. and $x+2$

So H.C.F $= a(x+2)$

Find the H.C.F of the following expresion by factorization

1. $(x-6)^2$, x^2-36

Soln: $(x-6)^2 = (x-6)(x-6)$

$x^2-36 = (x)^2-(6)^2 = (x+6)(x-6)$

As $(x-6)$ is common so

$H.C.F = x-6$

2. $a^2b^2(a^2+ab+b^2)$, $a^3b^3(a^3-b^3)$

Soln: $a^2b^2(a^2+ab+b^2) = aabb\,(a^2+ab+b^2)$

$a^3b^3(a^3-b^3) = aaabbb(a-b)(a^2+ab+b^2)$

$H.C.F = aabb\,(a^2+ab+b^2) = a^2b^2(a^2+ab+b^2)$

3. x^4-y^4, $x^4+2x^2y^2+y^4$

Soln: $x^4-y^4 = (x^2)^2-(y^2)^2 = (x^2+y^2)(x^2-y^2)$

$= (x^2+y^2)(x+y)(x-y)$

$x^4+2x^2y^2+y^4 = (x^2)^2+2(x^2)(y^2)+(y^2)^2$

$= (x^2+y^2)^2 = (x^2+y^2)(x^2+y^2)$

$\therefore\ H.C.F = (x^2+y^2)$

4. $6m^2+5m+4$ and $16m^2-16m+4$

Soln: $6m^2+5m-4 = 6m^2+8m-3m-4$

$= 2m(3m+4)-1(3m+4)$

$= (3m+4)(2m-1)$

$16m^2-16m+4 = 4\,(4m^2-4m+1)$

$= 4[(2m)^2-2(2m)(1)+(1)^2]$

$= 4(2m-1)^2 = 4(2m+1)(2m+1)$

$\therefore\ H.C.F = 2m-1$

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5. $2ax^2 - 2ax - 12a,\ 3a^2x^2 - 7a^2x - 6a^2,\ a^3x^2 + 4a^3x - 21a^3$

Soln:- $2ax^2 - 2ax - 12a$

$= 2a(x^2 - x - 6)$

$= 2a[x^2 - 3x + 2x - 6] = 2a[x(x-3) + 2(x-3)]$

$= 2a(x-3)(x+2)$

$3a^2x^2 - 7a^2x - 6a^2 = a^2[3x^2 - 7x - 6]$

$= a^2[3x^2 - 9x + 2x - 6] = a^2[3x(x-3) + 2(x-3)]$

$= a^2(3x+2)(x-3)$

$a^3x^2 + 4a^3x - 21a^3 = a^3[x^2 + 4x - 21]$

$= a^3[x^2 + 7x - 3x - 21]$

$= a^3[x(x+7) - 3(x+7)]$

$= a^3(x-3)(x+7)$

Since The common factors are a, $x-3$

So $H.C.F = a(x-3)$

Find The H.C.F of the following expressions by direct division method.

6. $x^2 - x - 6$ and $x^2 - 2x - 3$

Soln:

$x^2 - 2x - 3 \,)\, x^2 - x - 6 \,(\, 1$

$\pm x^2 \mp 2x \mp 3$

$x - 3 \,)\, x^2 - 2x - 3 \,(\, x + 1$

$\pm x^2 \mp 3x$

$x - 3$

$\pm x \mp 3$

So $H.C.F = x - 3$

(70)

7. y^3-3y+2, y^3-5y^2+7y-3

Soln:-

$y^3-3y+2 \,)\, y^3-5y^2+7y-3 \,(\, 1$

$\pm y^3 \quad \mp 3y \pm 2$

$-5y^2+10y-5$ Taking -5 common

$y^2-2y+1 \,)\, y^3-3y+2 \,(\, y$

$\pm y^3 \mp 2y^2 \pm y$

$2y^2-4y+2$ (taking 2 common)

$y^2-2y+1 \,)\, y^2-2y+1 \,(\, 1$

$\pm y^2 \mp 2y \pm 1$

H.C.F $= y^2-2y+1$

8. $2x^5-4x^4-6x$ and $x^5+x^4-3x^3-3x^2$

Soln:-

$x^5+x^4-3x^3-3x^2 \,)\, 2x^5-4x^4-6x \,(\, 2$

$\pm 2x^5 \pm 2x^4 \mp 6x^3 \mp 6x^2$

$-6x^4+6x^3+6x^2-6x$ ∵ Taking (-6) Common

$x^4-x^3-x^2+x \,)\, x^5+x^4-3x^3-3x^2 \,(\, x$

$\pm x^5 \mp x^4 \mp x^3 \pm x^2$

$2x^4-2x^3-4x^2$ Taking 2 common

$x^4-x^3-2x^2$

$x^4-x^3-2x^2 \,)\, x^4-x^3-x^2+x \,(\, 1$

$\pm x^4 \mp x^3 \mp 2x^2$

$x^2+x \,)\, x^4-x^3-2x^2 \,(\, x^2-2x$

$\pm x^4 \pm x^3$

$-2x^3-2x^2$

$-2x^3-2x^2$

H.C.F $= x^2+x$

(71)

9) $2x^3+10x^2+5x+25,\ x^3+5x^2-x-5$

Soln:

$$x^3+5x^2-x-5 \,\big)\, 2x^3+10x^2+5x+25 \,(\, 2$$
$$\underline{\pm 2x^3 \pm 10x^2 \mp 2x \mp 10}$$
$$7x+35$$

Taking 7 common: $7x+35 = 7(x+5)$

$$x+5 \,\big)\, x^3+5x^2-x-5 \,(\, x^2-1$$
$$\underline{\pm x^3 \pm 5x^2}$$
$$-x-5$$
$$\underline{\mp x \mp 5}$$
$$0$$

So, the required G.C.D $= x+5$

10) $x^3-x^2-2x,\ 2x^3+x^2-10x$ and x^3+2x^2-8x

Soln:

$$x^3-x^2-2x \,\big)\, 2x^3+x^2-10x \,(\, 2$$
$$\underline{\pm 2x^3 \mp 2x^2 \mp 4x}$$
$$3x^2-6x$$

Taking 3 common: $3x^2-6x = 3(x^2-2x)$

$$x^2-2x \,\big)\, x^3-x^2-2x \,(\, x+1$$
$$\underline{\pm x^3 \mp 2x^2}$$
$$x^2-2x$$
$$\underline{\pm x^2 \mp 2x}$$
$$0$$

Now

$$x^2-2x \,\big)\, x^3+2x^2-8x \,(\, x+4$$
$$\underline{\pm x^3 \mp 2x^2}$$
$$4x^2-8x$$
$$\underline{\pm 4x^2 \mp 8x}$$
$$0$$

So the required H.C.F $= x^2-2x$

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LEAST COMMON MULTIPLE (L.C.M)

The L.C.M of two or more polynomial is a polynomial of least degree which is divisible by the given polynomials. There are two methods for finding the L.C.M. i.e Factorization and division

(i) Factorization method:- The following formula is used for L.C.M

L.C.M = Common factors x Non Common factors

(ii) Division method:- For division method the formula which is used is given by

Product of polynomials = L.C.M x H.C.F

i.e $A \times B = L \times H$

$$\Rightarrow L = \frac{A \times B}{H}$$

Ex# 3.2

Find the L.C.M of the following by Factorization:

1. $x^3 - 8$, $x^2 + x - 6$

Soln:- $x^3 - 8 = (x)^3 - (2)^3 = (x-2)[x^2 + 2(x) + 2^2]$

$= (x-2)(x^2 + 2x + 4)$

$x^2 + x - 6 = x^2 + 3x - 2x - 6$

$= x(x+3) - 2(x+3) = (x+3)(x-2)$

$\therefore$ L.C.M = Common factor . Non common.

$= (x-2)(x+3)(x^2 + 2x + 4)$ Ans

2. x^2-5x+6 and x^2-4

Soln:- $x^2-5x+6 = x^2-2x-3x+6$

$= x(x-2)-3(x-2)$

$= (x-2)(x-3)$

$x^2-4 = (x)^2-(2)^2 = (x+2)(x-2)$

Common factors $= x-2$, Non common factors $= x-3, x+2$

So L.C.M = Product of C.F x Product of non factors

$= (x-2)(x+2)(x-3)$

$= (x^2-4)(x-3)$

3. x^2+3x+2, x^2+5x+6, x^2-2x-8

Soln:- $x^2+3x+2 = x^2+2x+x+2 = x(x+2)+1(x+2)$

$= (x+2)(x+1)$

$x^2+5x+6 = x^2+3x+2x+6 = x(x+3)+2(x+3)$

$= (x+3)(x+2)$

$x^2-2x-8 = x^2-4x+2x-8 = x(x-4)+2(x-4)$

$= (x-4)(x+2)$

Common Factors $= x+2$, Non common $= x-4, x+3, x+1$

L.C.M $= (x+2)(x-4)(x+3)(x+1)$

4. $2x^4+3x^2+1$, $5x^3-7x^2+5x-7$, x^3-1

Soln:- $2x^4+3x^2+1 = 2x^4+2x^2+x^2+1$

$= 2x^2(x^2+1)+1(x^2+1)$

$= (x^2+1)(2x^2+1)$

(74)

$5x^3-7x^2+5x-7 = x^2(5x-7)+1(5x-7)$

$= (5x-7)(x^2+1)$

$x^3-1 = (x)^3-(1)^3 = (x-1)[(x)^2+(x)(1)+(1)^2]$

$= (x-1)(x^2+x+1)$

C. Factors = x^2+1

Non common factors = $(x-1)(x^2+x+1)(5x-7)(2x^2+1)$

So required L.C.M $= (x^2+1)(x-1)(x^2+x+1)(5x-7)(2x^2+1)$

$= (x^2+1)(x^3-1)(5x-7)(2x^2+1)$

Find the L.C.M and H.C.F of the following

5. $2x^3-5x^2+4,\ x^3-16x+24$

Soln:-

$x^3-16x+24 \,\overline{)\, 2x^3-5x^2+4 \,}\ \ 2$

$\pm 2x^3 \mp 32x \pm 48$

$-1 \mid -5x^2+32x-44$

$5x^2-32x+44 \,\overline{)\, x^3-16x+24\,}\ \ x+8$ (Taking -1 common)

$\times 5$

$5x^3-80x+120$

$\pm 5x^3 \pm 44x \mp 32x^2$

Taking 4 common $\quad 4 \mid +32x^2-124x+120$

$8x^2-31x+30$

xing by 5 $\quad \times 5$

$40x^2-155x+150$

$\pm 40x^2 \mp 256x \pm 352$

Taking 101 common $\quad 101 \mid 101x-202$

$x-2$.

(75)

$$x-2\overline{)5x^2-32x+44}\,(5x-22$$
$$\underline{\pm 5x^2 \mp 10x}$$
$$-22x+44$$
$$\underline{\mp 22x \pm 44}$$

$\Rightarrow H.C.F = (x-2)$

Now we know that $L.C.M \cdot H.C.F = A \times B$

$$\Rightarrow L.C.M = \frac{A \times B}{H}$$

or $$L = \frac{(x^3-16x+24)(2x^3-5x^2+4)}{(x-2)}$$

$$x-2\overline{)x^3-16x+24}\,(x^2+2x-12$$
$$\underline{\pm x^3 \mp 2x^2}$$
$$2x^2-16x$$
$$\underline{\pm 2x^2 \mp 4x}$$
$$-12x+24$$
$$\underline{\mp 12x \pm 24}$$

So $L.C.M = (x^2+2x-12)(2x^3-5x^2+4)$

6. $x^3-2x^2-13x-10$ and $x^3-x^2-10x-8$

Soln:

$$x^3-x^2-10x-8\overline{)x^3-2x^2-13x-10}\,(1$$
$$\underline{\pm x^3 \mp x^2 \mp 10x \mp 8}$$
$$-1\,|\,-x^2-3x-2 \quad \text{Taking } -1 \text{ common}$$

$$x^2+3x+2\overline{)x^3-x^2-10x-8}\,(x-4$$
$$\underline{\pm x^3 \pm 3x^2 \pm 2x}$$
$$-4x^2-12x-8$$
$$\underline{\mp 4x^2 \mp 12x \mp 8}$$

(76)

So $H.C.F = x^2+3x+2$

But $L.C.M = \frac{A \times B}{H.C.F}$

$$\Rightarrow L.C.M = \frac{(x^3-2x^2-13x-10)(x^3-x^2-10x-8)}{(x^2+3x+2)}$$

x^2+3x+2) $x^3-2x^2-13x-10$ ($x-5$

$\quad x^3+3x^2+2x$

$\quad -5x^2-15x-10$

$\quad -5x^2-15x-10$

So $L.C.M = (x-5)(x^3-x^2-10x-8)$

7. a^4+a^2+1 and a^4-a^3-a+1

Sol: $a^4+a^2+1 = a^4+2a^2+1-a^2 = (a^2+1)^2-(a)^2$

$= (a^2+1+a)(a^2+1-a)$

$a^4-a^3-a+1 = a^3(a-1)-1(a-1)$

$= (a^3-1)(a-1) = (a-1)(a^2+a+1)(a-1)$

So $H.C.F = a^2+a+1$

$L.C.M = (a^2+a+1)(a-1)(a^2-a+1)(a-1)$

$= (a^4-a^3-a+1)(a^2-a+1)$

8. H.C.F and L.C.M of two polynomials $x-2$ and x^3+3x^2-6x-8 respectively. If one poly is x^2+2x+8, Then find the other polynomial.

(77)

Soln: Here, $H = x-2$

$L = x^3+3x^2-6x-8$, $A = x^2+2x-8$, $B=?$

As $A \times B = L \times H$

$\Rightarrow B = \frac{L \times H}{A}$ ÷ing both sides by A

Putting the values we get

$$B = \frac{(x^3+3x^2-6x-8)(x-2)}{(x^2+2x-8)} \quad \text{---} ①$$

Now

$$\begin{array}{r|l} & x+1 \\ x^2+2x-8 & x^3+3x^2-6x-8 \\ & \underline{\pm x^3 \pm 2x^2 \mp 8x} \\ & x^2+2x-8 \\ & \underline{\pm x^2 \pm 2x \mp 8} \\ & 0 \end{array}$$

So ① $\Rightarrow B = (x+1)(x-2) = x^2-x-2$

9) If the product of two polynomials is $x^4+6x^3-3x^2-56x-48$ and the L.C.M is $x^3+2x^2-11x-12$, then find the H.C.F

Soln:- Here;

$A \times B = x^4+6x^3-3x^2-56x-48$

$L.C.M = x^3+2x^2-11x-12$, $H.C.F = ?$

$\because L.C.M \times H.C.F = A \times B$

$\Rightarrow H.C.F = \frac{A \times B}{L.C.M}$

$= \frac{x^4+6x^3-3x^2-56x-48}{x^3+2x^2-11x-12}$

$$x^3+2x^2-11x-12 \overline{\big)\, x^4+6x^3-3x^2-56x-48 \,}\; x+4$$
$$\underline{\pm x^4 \pm 2x^3 \mp 11x^2 \mp 12x}$$
$$4x^3+8x^2-44x-48$$
$$\underline{\pm 4x^3 \pm 8x^2 \mp 44x \mp 48}$$

So $\dfrac{x^4+6x^3-3x^2-56x-48}{x^3+2x^2-11x-12} = x+4$

Hence, HCF $= x+4$

10. H.C.F and L.C.M of two polynomials are $x+5$ and $2x^3+11x^2+2x-15$ respectively. Find the polynomials of degree 2.

Soln :- Given that $H = x+5$, $L = 2x^3+11x^2+2x-15$

$A = ?$ $B = ?$

As $A \times B = H \times L = (x+5)(2x^3+11x^2+2x-15)$ —(i)

Now we will factorize the cubic poly using remainder theorem

$P(x) = 2x^3+11x^2+2x-15$ —— (ii)

Putting $x = 1$ in (ii)

$P(1) = 2(1)^3+11(1)^2+2(1)-15$

$P(1) = 2+11+2-15 = 15-15 = 0$

$\Rightarrow x = 1$ is root of $P(x)$

$\Rightarrow x-1$ is factor of $P(x)$

For other factors $\div 2x^3+11x^2+2x-15$ by $x-1$

$$
\begin{array}{r|l}
 & 2x^2+13x+15 \\
x-1 & 2x^3+11x^2+2x-15 \\
 & \underline{\pm 2x^3 \mp 2x^2} \\
 & 13x^2+2x \\
 & \underline{\pm 13x^2 \mp 13x} \\
 & 15x-15 \\
 & \underline{\pm 15x \mp 15} \\
\end{array}
$$

So $2x^3+11x^2+2x-15=(x-1)(2x^2+13x+15)$

Putting this in Eq (1) we get

$$A \times B = (x+5)(x-1)(2x^2+13x+15)$$

$$= (x^2-x+5x-5)(2x^2+13x+15)$$

$$= (x^2+4x-5)(2x^2+13x+15)$$

$$\Rightarrow A = x^2+4x-5,\ B = 2x^2+13x+15$$

(80)

Ex# 3.3

Simplify the following expressions.

1. $\frac{2x}{x-y} - \frac{2y}{x-y}$

Soln:- $\frac{2x}{x-y} - \frac{2y}{x-y} = \frac{2x-2y}{x-y} = \frac{2(x-y)}{(x-y)} = 2.$

2. $\frac{x+2}{x+3} - \frac{x-1}{x-3} + \frac{4x}{x^2-9}$

Soln $\frac{x+2}{x+3} - \frac{x-1}{x-3} + \frac{4x}{x^2-9}$

$= \left[\frac{(x+2)(x-3)-(x-1)(x+3)}{(x+3)(x-3)}\right] + \frac{4x}{x^2-9}$

$= \left[\frac{x^2-3x+2x-6-(x^2+3x-x-3)}{x^2-9}\right] + \frac{4x}{x^2-9}$

$= \frac{x^2-x-6-x^2-2x+3}{x^2-9} + \frac{4x}{x^2-9}$

$= \frac{-3x-3}{x^2-9} + \frac{4x}{x^2-9} = \frac{-3x-3+4x}{x^2-9}$

$= \frac{(x-3)}{(x+3)(x-3)} = \frac{1}{x+3}$ Ans

3. $\frac{a}{3x-y} + \frac{a}{3x+y} - \frac{6ax}{9x^2-y^2}$

Soln:- $\frac{a}{3x-y} + \frac{a}{3x+y} - \frac{6ax}{9x^2-y^2}$

$= \left(\frac{a}{3x-y} + \frac{a}{3x+y}\right) - \frac{6ax}{9x^2-y^2}$

(81)

$$= \left[\frac{a(3x+y)+a(3x-y)}{(3x-y)(3x+y)}\right] - \frac{6ax}{9x^2-y^2}$$

$$= \frac{3ax+ay+3ax-ay}{(3x)^2-(y)^2} - \frac{6ax}{9x^2-y^2}$$

$$= \frac{6ax}{9x^2-y^2} - \frac{6ax}{9x^2-y^2} = \frac{6ax-6ax}{9x^2-y^2}$$

$$= \frac{0}{9x^2-y^2} = 0$$

4. $\frac{1}{a-b} + \frac{1}{a+b} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$

Soln:- $\left(\frac{1}{a-b} + \frac{1}{a+b}\right) + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$

$$= \left(\frac{a+b+a-b}{(a-b)(a+b)}\right) + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$$

$$= \left(\frac{2a}{a^2-b^2} + \frac{2a}{a^2+b^2}\right) + \frac{4a^3}{a^4+b^4}$$

$$= \frac{2a(a^2+b^2)+2a(a^2-b^2)}{(a^2-b^2)(a^2+b^2)} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{2a^3+2ab^2+2a^3-2ab^2}{a^4-b^4} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{4a^3}{a^4-b^4} + \frac{4a^3}{a^4+b^4} = \frac{4a^3(a^4+b^4)+4a^3(a^4-b^4)}{(a^4-b^4)(a^4+b^4)}$$

$$= \frac{4a^7+4a^3b^4+4a^7-4a^3b^4}{a^8-b^8} = \frac{8a^7}{a^8-b^8}$$

(82)

5. $\frac{y}{x-y}+\frac{y}{x+y}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$

Soln: $\frac{y}{x-y}+\frac{y}{x+y}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$

$$=\frac{y(x+y)+y(x-y)}{(x+y)(x-y)}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$$

$$=\frac{xy+y^2+xy-y^2}{x^2-y^2}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$$

$$=\frac{2xy}{x^2-y^2}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$$

$$=\frac{2xy(x^2+y^2)+2xy(x^2-y^2)}{(x^2-y^2)(x^2+y^2)}+\frac{4x^3y}{x^4+y^4}$$

$$=\frac{2x^3y+2xy^3+2x^3y-2xy^3}{x^4-y^4}+\frac{4x^3y}{x^4+y^4}$$

$$=\frac{4x^3y}{x^4-y^4}+\frac{4x^3y}{x^4+y^4}=\frac{4x^3y(x^4+y^4)+4x^3y(x^4-y^4)}{(x^4-y^4)(x^4+y^4)}$$

$$=\frac{4x^7y+4x^3y^5+4x^7y-4x^3y^5}{x^8-y^8}$$

$$=\frac{8x^7y}{x^8-y^8}$$

6. $\frac{1}{a^2+7a+10}+\frac{1}{a^2+10a+16}$

Soln. $\frac{1}{a^2+7a+10}+\frac{1}{a^2+10a+16}$

(83)

$$= \frac{1}{a^2+2a+5a+10} + \frac{1}{a^2+2a+8a+16}$$

$$= \frac{1}{a(a+2)+5(a+2)} + \frac{1}{a(a+2)+8(a+2)} = \frac{1}{(a+2)(a+5)} + \frac{1}{(a+2)(a+8)}$$

$$= \frac{1}{a+2}\left[\frac{1}{a+5} + \frac{1}{a+8}\right] = \frac{1}{a+2}\left[\frac{a+8+a+5}{(a+5)(a+8)}\right]$$

$$= \frac{1}{a+2}\left(\frac{2a+13}{(a+5)(a+8)}\right) = \frac{2a+13}{(a+2)(a+5)(a+8)}$$

7. $\frac{4}{x+2} + \frac{8}{x+5} - \frac{8}{x+3} - \frac{4}{x+6}$

Soln:- $\frac{4}{x+2} + \frac{8}{x+5} - \frac{8}{x+3} - \frac{4}{x+6}$

$$= \left[\frac{4}{x+2} + \frac{8}{x+5}\right] - \left[\frac{8}{x+3} + \frac{4}{x+6}\right]$$

$$= \left[\frac{4(x+5)+8(x+2)}{(x+2)(x+5)}\right] - \left[\frac{8(x+6)+4(x+3)}{(x+3)(x+6)}\right]$$

$$= \frac{4x+20+8x+16}{(x+2)(x+5)} - \frac{8x+48+4x+12}{(x+3)(x+6)}$$

$$= \frac{12x+36}{(x+2)(x+5)} - \frac{12x+60}{(x+3)(x+6)}$$

$$= \frac{12(x+3)}{(x+2)(x+5)} - \frac{12(x+5)}{(x+3)(x+6)}$$

$$= 12\left[\frac{x+3}{(x+2)(x+5)} - \frac{x+5}{(x+3)(x+6)}\right]$$

$$= 12\left[\frac{(x+3)(x+3)(x+6) - (x+5)(x+5)(x+2)}{(x+2)(x+5)(x+3)(x+6)}\right]$$

$$= 12\left[\frac{(x+3)^2(x+6) - (x+5)^2(x+2)}{(x+2)(x+5)(x+3)(x+6)}\right]$$

$$= 12\left[\frac{(x+6)(x^2+6x+9) - (x+2)(x^2+10x+25)}{(x+2)(x+5)(x+3)(x+6)}\right]$$

$$= 12\left[\frac{x^3+6x^2+9x+6x^2+36x+54-(x^3+10x^2+25x+2x^2+20x+50)}{(x+2)(x+5)(x+3)(x+6)}\right]$$

$$= 12\left[\frac{x^3+12x^2+45x+54-x^3-12x^2-45x-50}{(x+2)(x+5)(x+3)(x+6)}\right]$$

$$= \frac{12(4)}{(x+2)(x+5)(x+3)(x+6)} = \frac{48}{(x+2)(x+5)(x+3)(x+6)}$$

Ex: 3.4

Simplify the following expressions:

1) $\frac{x^2-16}{x+4}$

Soln: $\frac{x^2-16}{x+4} = \frac{(x)^2-(4)^2}{x+4} = \frac{(x+4)(x-4)}{(x+4)} = x-4$

2. $\frac{5}{x-2} \times \frac{x-2}{30(x-1)}$

Soln: $\frac{5}{(x-2)} \times \frac{(x-2)}{30(x-1)}$

$$= \frac{1}{30(x-1)} = \frac{1}{6(x-1)}$$

(85)

3. $\frac{x^3-8}{2x^2} \times \frac{4x}{x^2-5x+6}$

Soln: $\frac{x^3-8}{2x^2} \times \frac{4x}{x^2-5x+6}$

$= \frac{(x)^3-(2)^3}{2x^2} \times \frac{4x}{x^2-2x-3x+6}$

$= \frac{(x-2)(x^2+2x+4)}{2\cdot x\cdot x} \times \frac{4x}{x(x-2)-3(x-2)}$

$= \frac{(x-2)(x^2+2x+4)}{2x\cdot x} \times \frac{^2 4x}{(x-2)(x-3)}$

$= \frac{2(x^2+2x+4)}{x(x-3)}$

4. $\frac{x^3+y^3}{x^2-y^2} \times \frac{x^2+xy+xz+yz}{x^4+x^2y^2+y^4} \times \frac{x^3-y^3}{x^3+y^3}$

Soln $\frac{x^3+y^3}{x^2-y^2} \times \frac{x(x+y)+z(x+y)}{x^4+y^4+x^2y^2} \times \frac{x^3-y^3}{x^3+y^3}$

$= \frac{1}{(x+y)(x-y)} \times \frac{(x+z)(x+y)}{x^4+y^4+2x^2y^2-x^2y^2} \times (x-y)(x^2+y^2+xy)$

$= \frac{(x+z)(x^2+y^2+xy)}{(x^2+y^2)^2-(xy)^2}$

$= \frac{(x+z)(x^2+y^2+xy)}{(x^2+y^2+xy)(x^2+y^2-xy)} = \frac{x+z}{x^2+y^2-xy}$ Ans

5. $\dfrac{x+2}{5(x-3)} \div \dfrac{x+2}{15(x-3)}$

Soln:- $\dfrac{x+2}{5(x-3)} \div \dfrac{x+2}{15(x-3)} = \dfrac{(x+2)}{5(x-3)} \times \dfrac{15(x-3)}{(x+2)}$

$= 3$

6. $\dfrac{a^2+b^2}{a^2+ab+b^2} \div \dfrac{a^4-b^4}{a^3-b^3}$

Soln:- $\dfrac{a^2+b^2}{a^2+ab+b^2} \div \dfrac{a^4-b^4}{a^3-b^3}$

$= \dfrac{a^2+b^2}{a^2+ab+b^2} \div \dfrac{(a^2)^2-(b^2)^2}{(a-b)(a^2+ab+b^2)}$

$= \dfrac{a^2+b^2}{a^2+ab+b^2} \div \dfrac{(a^2+b^2)(a^2-b^2)}{(a-b)(a^2+ab+b^2)}$

$= \dfrac{a^2+b^2}{a^2+ab+b^2} \times \dfrac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a+b)(a-b)} = \dfrac{1}{a+b}$

7. $\dfrac{x^2-x-6}{x^2+6x+9} \div \dfrac{x^2-4}{x+3}$

Soln:- $\dfrac{x^2-x-6}{x^2+6x+9} \div \dfrac{x^2-4}{x+3}$

$= \dfrac{x^2-3x+2x-6}{x^2+3x+3x+9} \times \dfrac{x+3}{(x)^2-(2)^2}$

$= \dfrac{x(x-3)+2(x-3)}{x(x+3)+3(x+3)} \times \dfrac{x+3}{(x+2)(x-2)}$

$= \dfrac{(x-3)(x+2)}{(x+3)(x+3)} \cdot \dfrac{(x+3)}{(x+2)(x-2)} = \dfrac{x-3}{(x+2)(x+3)}$

8. $\dfrac{a^4-8a}{2a^2+5a-3} \times \dfrac{2a-1}{a^2+2a+4} \div \dfrac{a^2-2a}{a+3}$

Soln:- $\dfrac{a^4-8a}{2a^2+5a-3} \times \dfrac{2a-1}{a^2+2a+4} \times \dfrac{a+3}{a^2-2a}$

$= \dfrac{a(a^3-8)}{2a^2+6a-a-3} \times \dfrac{2a-1}{a^2+2a+4} \times \dfrac{a+3}{a(a-2)}$

$= \dfrac{a(a-2)(a^2+2a+4)}{2a(a+3)-1(a+3)} \times \dfrac{2a-1}{a^2+2a+4} \times \dfrac{a+3}{a(a-2)}$

$= \dfrac{a(a-2)(a^2+2a+4)}{(a+3)(2a-1)} \times \dfrac{2a-1}{a^2+2a+4} \times \dfrac{a+3}{a(a-2)}$

$= 1$

Ex : 3.5

Find the Square root of the following by factorization.

1. $4x^2-20xy+25y^2$

Soln:- $4x^2-20xy+25y^2 = (2x)^2-2(2x)(5y)+(5y)^2$

$= (2x-5y)^2$

Taking Sq. root on both sides we get.

$\sqrt{4x^2-20xy+25y^2} = \sqrt{(2x-5y)^2}$

$= \pm(2x-5y)$

(87)

2. $16x^2+72xy+81y^2$

Soln:- $16x^2+72xy+81y^2 = (4x)^2+2(4x)(9y)+(9y)^2$

$= (4x+9y)^2$

Taking square root of both sides

$\sqrt{16x^2+72xy+81y^2} = \sqrt{(4x+9y)^2}$

$= \pm(4x+9y)$

3. $(x-y)^2+6(x-y)+9$

Soln:- Let $x-y=A$ then

$(x-y)^2+6(x-y)+9 = A^2+6A+9$

$=(A)^2+2(A)(3)+3^2 = (A+3)^2$

Taking Sq-root on both sides we get

$\sqrt{(x-y)^2+6(x-y)+9} = \sqrt{(A+3)^2} = \pm(A+3)$

$= \pm(x-y+3)$ ∵ Putting $A=x-y$

4. $\left(x^2+\frac{1}{x^2}\right)^2-4(x-1/x)^2-4$

Soln:- $\left(x^2+\frac{1}{x^2}\right)^2-4\left(x-\frac{1}{x}\right)^2-4$

$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}-2\right)-4$

$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)+8-4$

$=\left(x^2+1/x^2\right)^2-4(x^2+1/x^2)+4$

(89)

Putting $x^2+\frac{1}{x^2} = y$ Then we have

$(x^2+1/x^2)^2 - 4(x^2+\frac{1}{x^2})+4 = y^2-4y+4$

$= (y)^2 - 2(y)(2) + (2)^2$

$= (y-2)^2$

Taking Square root on both sides we get

$\sqrt{(x^2+\frac{1}{x^2})^2 - 4(x^2+\frac{1}{x^2})+4} = \sqrt{(y-2)^2} = \pm(y-2)$

$= \pm(x^2+1/x^2-2) \quad \because y = x^2+1/x^2$

5. $x^2+\frac{1}{x^2} - 8(x+\frac{1}{x})+18$

Soln:- $x^2+\frac{1}{x^2} - 8(x+\frac{1}{x})+8 = x^2+\frac{1}{x^2}+2-8(x+\frac{1}{x})+16$

$= (x+\frac{1}{x})^2 - 8(x+\frac{1}{x})+16$

Putting $x+\frac{1}{x} = y$ in R.H.S we get.

$x^2+\frac{1}{x^2} - 8(x+\frac{1}{x})+8 = y^2-8y+16 = (y-4)^2$

Taking Square root on both sides we get

$\sqrt{x^2+\frac{1}{x^2} - 8(x+\frac{1}{x})+8} = \sqrt{(y-4)^2}$

$= (y-4)$

$= (x+\frac{1}{x}-4) \quad \because y = x+\frac{1}{x}$

(90)

Find the square root by division method.

6. $x^6 - 2x^5 + 3x^4 - 2x^3 + x^2$

Soln.

Quotient: $x^3 - x^2 + x$

x^3 | $x^6 - 2x^5 + 3x^4 - 2x^3 + x^2$
$\pm x^6$

$2x^3 - x^2$ | $-2x^5 + 3x^4$
$\mp 2x^5 \pm x^4$

$2x^3 - 2x^2 + x$ | $2x^4 - 2x^3 + x^2$
$\pm 2x^4 \mp 2x^3 \pm x^2$

So $\sqrt{x^6 - 2x^5 + 3x^4 - 2x^3 + x^2} = \pm(x^3 - x^2 + x)$

7. $\frac{x^4}{4} + 4x^2 + \frac{ax^2}{3} + \frac{a^2}{9} - 2x^3 - \frac{4ax}{3}$

Soln:

Quotient: $x^2/2 - 2x + a/3$

$x^2/2$ | $\frac{x^4}{4} - 2x^3 + 4x^2 + \frac{ax^2}{3} - \frac{4ax}{3} + \frac{a^2}{9}$
$\pm \frac{x^4}{4}$

$x^2 - 2x$ | $-2x^3 + 4x^2$
$\mp 2x^3 \pm 4x^2$

$x^2 - 4x + \frac{a}{3}$ | $\frac{ax^2}{3} - \frac{4ax}{3} + \frac{a^2}{9}$
$\pm \frac{ax^2}{3} \mp \frac{4ax}{3} \pm \frac{a^2}{9}$

(91)

Therefore, $\sqrt{\frac{x^4}{4}+4x^2+\frac{ax^2}{3}-2x^3-\frac{4ax}{3}+\frac{a^2}{9}} = \pm\left(\frac{x^2}{2}-2x+\frac{a}{3}\right)$

8. $x^4+x^3-\frac{31}{4}x^2-4x+16$

Soln:

$$
\begin{array}{l|l}
 & x^2+\frac{x}{2}-4 \\
x^2 & x^4+x^3-\frac{31}{4}x^2-4x+16 \\
 & \pm x^4 \\
\hline
2x^2+\frac{x}{2} & x^3-\frac{31}{4}x^2 \\
 & \pm x^3 \pm\frac{x^2}{4} \\
\hline
2x^2+x-4 & -8x^2-4x+16 \\
 & -8x^2 \mp 4x \pm 16 \\
\hline
\end{array}
$$

$-\frac{31}{4}x^2-\frac{x^2}{4} = \frac{-31x^2-x^2}{4} = \frac{-32x^2}{4} = -8x^2$

Therefore; $\sqrt{x^4+x^3-\frac{31}{4}x^2-4x+16} = \pm\left(x^2+\frac{x}{2}-4\right)$

9. For what value of K, the expression $4x^4+32x^2+96+\frac{128}{x^2}+\frac{K}{x^4}$ will become a perfect Square.

Soln:-

$$
\begin{array}{l|l}
 & 2x^2+8+\frac{8}{x^2} \\
2x^2 & 4x^4+32x^2+96+\frac{128}{x^2}+\frac{K}{x^4} \\
 & \pm 4x^4 \\
\hline
4x^2+8 & 32x^2+96 \\
 & \pm 32x^2 \pm 64 \\
\hline
4x^2+16+\frac{8}{x^2} & 32+\frac{128}{x^2}+\frac{K}{x^4} \\
 & \pm 32 \pm\frac{128}{x^2} \pm\frac{64}{x^4} \\
\hline
\end{array}
$$

$\frac{K}{x^4}-\frac{64}{x^4}$

Given that the expression is perfect square. So

$$\frac{K}{x^4} - \frac{64}{x^4} = 0$$

$\Rightarrow K - 64 = 0$ xing both sides by x^4

$\Rightarrow K = 64$

Ex: 3.6 (Objective Type Questions)

1) Encircle 'T' for true and 'F' for false.

(i) H.C.F of x^2-9 and $(x+3)$ is $x+3$. (T)

(ii) H.C.F of $a^3-\frac{1}{8}$ and $a^2+\frac{1}{2}a+\frac{1}{4}$ is $a-\frac{1}{2}$. (F)

(iii) L.C.M of a^4-b^4 and a^2-b^2 is a^2+b^2. (F)

(iv) L.C.M of x^3+8 and $x+2$ is x^3+8. (T)

(v) H.C.F of x^2+3x+2 and x^2+4x+3 is $x+1$. (T)

2. Fill in the blanks:

(i) H.C.F of x^2-5x+6 and $x-2$ is $\underline{x-2}$.

(ii) L.C.M of a^2+3a+2 and a^2+4a+3 is $\underline{(a+1)(a+2)(a+3)}$

(iii) H.C.F of $x^3-\frac{1}{x^3}$ and $x-\frac{1}{x}$ is $\underline{x-\frac{1}{x}}$

(iv) L.C.M is the $\underline{\text{Least}}$ degree polynomial which is divisible by the given polynomials

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(v) H.C.F is the highest degree polynomial which divides Completely the given polynomials.

3. Write the correct option in the box.

Soln (i) b (ii) a (iii) c (iv) c

(v) a (vi) a (vii) d (viii) c

Ex: 3.7

1. Find the H.C.F of x^3+4x^2+4x+3 and x^3+2x^2-4x-3 by division method.

Soln:

$$x^3+2x^2-4x-3 \,\big)\, x^3+4x^2+4x+3 \,\big(\, 1$$
$$\underline{\pm x^3 \pm 2x^2 \mp 4x \mp 3}$$
$$2x^2+8x+6$$

Taking 2 Common

$$2\,|\,2x^2+8x+6$$

$$x^2+4x+3 \,\big)\, x^3+2x^2-4x-3 \,\big(\, x-2$$
$$\underline{\pm x^3 \pm 4x^2 \pm 3x}$$
$$-2x^2-7x-3$$
$$\underline{\mp 2x^2 \mp 8x \mp 6}$$

$$x+3 \,\big)\, x^2+4x+3 \,\big(\, x+1$$
$$\underline{\pm x^2 \pm 3x}$$
$$x+3$$
$$\underline{\pm x \pm 3}$$

So $H.C.F = x+3$

2. Find the L.C.M of $x^3+6x^2+11x+6$ and x^3+3x^2-x-3

Soln:- $x^3+3x^2-x-3 = x^2(x+3)-1(x+3)$

$= (x^2-1)(x+3) = (x+1)(x-1)(x+3)$

Now we will factorize $P(x) = x^3+6x^2+11x+6$ by Factor Th.

$\because P(-1) = (-1)^3+6(-1)^2+11(-1)+6 = -1+6-11+6 = 12-12 = 0$

$\Rightarrow x+1$ is factor of $P(x)$.

$\therefore$

$$\begin{array}{r|l} & x^2+5x+6 \\ \hline x+1 & x^3+6x^2+11x+6 \\ & \pm x^3 \pm x^2 \\ \hline & 5x^2+11x \\ & \pm 5x^2 \pm 5x \\ \hline & 6x+6 \\ & \pm 6x \pm 6 \\ \hline & \end{array}$$

$\therefore x^3+6x^2+5x+6 = (x+1)(x^2+5x+6) = (x+1)(x^2+2x+3x+6)$

$= (x+1)[x(x+2)+3(x+2)] = (x+1)(x+2)(x+3)$

Therefore L.C.M $= (x+1)(x+3)(x-1)(x+2)$

$= (x+1)(x-1)(x+3)(x+2) = (x^2-1)(x^2+5x+6)$

3. The Product of two polynomials is $x^4+5x^3-6x^2-2x-28$ whose H.C.F is $x-2$, Then find the L.C.M

Soln:- As we know that $A \times B = L \times H$

$\Rightarrow L.C.M = \frac{A \times B}{H}$

$\Rightarrow L = \frac{(x^4+5x^3-6x^2-2x-28)}{(x-2)}$

$= x^3+7x^2+8x+14$

$$\begin{array}{r|l} & x^3+7x^2+2x+14 \\ \hline x-2 & x^4+5x^3-6x^2-2x-28 \\ & \pm x^4 \mp 2x^3 \\ \hline & 7x^3-6x^2 \\ & \pm 7x^3 \mp 14x^2 \\ \hline & 8x^2-2x \\ & \pm 8x^2 \mp 16x \\ \hline & 14x-28 \\ & 14x-28 \end{array}$$

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4. Simplify: $\frac{x-5}{x^2-7x+10} - \frac{2(x-6)}{x^2-9x+18} + \frac{x-7}{x^2-4x-21}$

Soln: $\frac{x-5}{x^2-7x+10} - \frac{2(x-6)}{x^2-9x+18} + \frac{x-7}{x^2-4x-21}$

$= \frac{x-5}{x^2-2x-5x+10} - \frac{2(x-6)}{x^2-6x-3x+18} + \frac{x-7}{x^2-7x+3x-21}$

$= \frac{x-5}{x(x-2)-5(x-2)} - \frac{2(x-6)}{x(x-6)-3(x-6)} + \frac{x-7}{x(x-7)+3(x-7)}$

$= \frac{(x-5)}{(x-2)(x-5)} - \frac{2(x-6)}{(x-3)(x-6)} + \frac{(x-7)}{(x-7)(x+3)}$

$= \frac{1}{x-2} - \frac{2}{x-3} + \frac{1}{x+3}$

$= \frac{x-3-2(x-2)}{(x-2)(x-3)} + \frac{1}{x+3}$ Taking L.C.M

$= \frac{x-3-2x+4}{(x-2)(x-3)} + \frac{1}{x+3}$

$= \frac{-x+1}{(x-2)(x-3)} + \frac{1}{x+3} = \frac{(-x+1)(x+3)+(x-2)(x-3)}{(x-2)(x+3)(x-3)}$

$= \frac{-x^2-3x+x+3+x^2-3x-2x+6}{(x-2)(x+3)(x-3)}$

$= \frac{-7x+9}{(x-2)(x+3)(x-3)}$

5. Simplify: $\frac{x^3-8}{2x^2} \times \frac{4x}{x^2-5x+6}$

Soln: $\frac{x^3-8}{2x^2} \times \frac{4x}{x^2-5x+6} = \frac{(x)^3-(2)^3}{2x^2} \times \frac{4x}{x^2-2x-3x+6}$

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$= \dfrac{(x-2)(x^2+2x+4)}{2x^2} \times \dfrac{4x}{x(x-2)-3(x-2)}$

$= \dfrac{(x-2)(x^2+2x+4)}{2x^2} \times \dfrac{4x}{(x-2)(x-3)} = \dfrac{2(x^2+2x+4)}{x(x-3)}$

6) Simplify: $\dfrac{5}{x-1} \div \dfrac{25(x-2)}{x-1}$

Soln:- $\dfrac{5}{x-1} \div \dfrac{25(x-2)}{x-1} = \dfrac{5}{(x-1)} \times \dfrac{(x-1)}{25(x-2)}$

$= \dfrac{1}{5(x-2)}$

7) Find the Sq root of $(x^2+6x+5)(x^2+9x+8)(x^2+13x+40)$ by factorization.

Soln

$x^2+6x+5 = x^2+5x+x+5 = x(x+5)+1(x+5)$
$= (x+1)(x+5)$

$x^2+9x+8 = x^2+8x+x+8 = x(x+8)+1(x+8)$
$= (x+8)(x+1)$

$x^2+13x+40 = x^2+5x+8x+40 = x(x+5)+8(x+5)$
$= (x+8)(x+5)$

$\therefore (x^2+6x+5)(x^2+9x+8)(x^2+13x+40) = (x+1)(x+5)(x+8)(x+1)(x+8)(x+5)$
$= (x+1)^2(x+5)^2(x+8)^2$

Taking the squareroot on both sides we get

$\sqrt{(x^2+6x+5)(x^2+9x+8)(x^2+13x+40)} = \sqrt{(x+1)^2(x+5)^2(x+8)^2}$
$= \pm (x+1)(x+5)(x+8)$

End of Chap # 3

Unit 4 LINEAR EQUATIONS AND INEQUALITIES

Linear Equation :- The equation of the form $ax+b$, $a\neq 0$ is called linear equation where $a, b \in Q$.

The value of x which satisfy the equation is called its solution. e.g. $3x-4=0$, $\frac{1}{2}x+8=0$ etc.

Exp 1 :- Solve the equation: $15x-6 = 3x+18$

Soln :

$15x-6 = 3x+18$

$15x-6+6 = 3x+18+6$ adding 6 to both sides

$15x-3x = 3x-3x+24$ Sub $3x$ " "

$12x = 24$

$\frac{12x}{12} = \frac{24}{12}$ ÷ing b.s by 12

$\Rightarrow x = 2$

Verification: Putting $x=2$ in $15x-6=3x+18$

$15(2)-6 = 3(2)+18$

$30-6 = 6+18$

$24 = 24$ which is true

$\therefore$ S.S $= \{2\}$

Exp 3 :- A number consists of two digits. The sum of digits is 8. If the digits are interchanged, then the new number becomes 36 less than the original number. Find the number.

Soln: Let the digit at unit place $= x$

" " " " ten's place $= y$

Then according to 1st condition: $x+y = 8$ ——— (1)

(98)

he orignal number $= x+10y$

After changing the place, The new number $= y+10x$

ccording to condition 2: New number become 36 less

i.e $x+10y = y+10x+36$

$\Rightarrow x-10x+10y-y=36$

$-9x+9y=36$

$\div$ ing by (+9) we get $-x+y=4$ — (ii)

adding (i) and (ii)

$$\begin{array}{r} x+y=8 \\ -x+y=4 \\ \hline 2y=12 \end{array} \Rightarrow \frac{2y}{2}=\frac{12}{2} \Rightarrow \boxed{y=6}$$

Putting $y=6$ in (ii) $\Rightarrow -x+6=4 \Rightarrow -x=4-6=-2$

$\Rightarrow \boxed{x=2}$

So the required number $= 62$

EX: 4.1

Find the solution set of the equation and also verify it.

(1) $3x-5=7$

Soln:- $3x-5=7$

$3x-5+5=7+5$ by adding 5 to both sides

$3x=12$ $\div$ ing both sides by 3

$\frac{3x}{3}=\frac{12}{3}$

$\Rightarrow x=4$

Varification:- Putting $x=4$ in $3x-5=7$

$\Rightarrow 3(4)-5=7$

$12-5=7$

$7=7$ which is true. Hence $SS=\{4\}$

(99)

2. $4(x+2)-2(x-2)=18$

Soln:- $4(x+2)-2(x-2)=18$

$4x+8-2x+4=18$

$(4x-2x)+(8+4)=18$

$2x+12=18$

$\Rightarrow 2x=18-12=6$

$\Rightarrow \frac{2x}{2}=\frac{6}{2}$

$\Rightarrow x=3$

$\therefore S.S=\{3\}$

Varification:

Putting $x=3$ in

$4(x+2)-2(x-2)=18$

$4(3+2)-2(3-2)=18$

$4(5)-2(1)=18$

$20-2=18$

$18=18$

which is true.

3. $6x+3=2x+19$

Soln:- $6x+3=2x+19$

$\Rightarrow 6x-2x+3=19$

$4x=19-3$

$\Rightarrow 4x=16$

$\frac{4x}{4}=\frac{16}{4}$ ÷ by 4

$\Rightarrow x=4$

Varifaction

Putting $x=4$ in

$6x+3=2x+19$

$6(4)+3=2(4)+19$

$24+3=8+19$

$27=27$

Which is true. Hence

$S.S=\{4\}$

4. $\frac{1}{x-2}=\frac{2}{x-1}$

Soln $\frac{1}{x-2}=\frac{2}{x-1}$

xing both sides by $(x-2)(x-1)$

$(x-2)(x-1)\frac{1}{x-2}=(x-2)(x-1)\cdot\frac{2}{x-1}$

$x-1=(x-2)2$

$\Rightarrow x-1=2x-4$

$\Rightarrow x-2x=-4+1$

$\Rightarrow -x=-3$

$\Rightarrow x=+3$

Varification.

Putting $x=3$ in

$\frac{1}{x-2}=\frac{2}{x-1}$

$\frac{1}{3-2}=\frac{2}{3-1}$

$\frac{1}{1}=\frac{2}{2}=1$ True

So $S.S=\{3\}$

(100)

5. $\frac{1}{7x+13} = \frac{2}{19}$

Soln $\frac{1}{7x+13} = \frac{2}{19}$

$\Rightarrow 2(7x+13) = 19$ by cross multiplication

$14x + 26 = 19$

$\Rightarrow 14x = 19 - 26$ Subtracting 26 from b.s

$14x = -7$

$\frac{14x}{14} = -7/14$ ÷ ing by 14

$x = -7/14 = -1/2$

$\therefore S.S = \{-1/2\}$

6. $\frac{x+5}{3} - \left(\frac{2-x}{2} - \frac{1}{4}\right) = \frac{3x+25}{12}$ — (i)

Soln:- $\frac{x+5}{3} - \frac{2-x}{2} + \frac{1}{4} = \frac{3x+25}{12}$

xing both sides by 12 we get

$12\frac{(x+5)}{3} - 12\left(\frac{2-x}{2}\right) + 12\cdot\frac{1}{4} = 12\left(\frac{3x+25}{12}\right)$

$4x + 20 - 12 + 6x + 3 = 3x + 25$

$10x + 11 = 3x + 25$

$10x - 3x = 25 - 11$

$7x = 14 \Rightarrow \frac{7x}{7} = \frac{14}{7} \Rightarrow x = 2$

Varification Putting $x = 2$ in (i)

$\frac{x+5}{3} - \left(\frac{2-x}{2} - \frac{1}{4}\right) = \frac{3x+25}{12}$ $\therefore S.S = \{2\}$

$\frac{2+5}{3} - \left(\frac{2-2}{2} - \frac{1}{4}\right) = \frac{6+25}{12}$

$\frac{7}{3} + 1/4 = \frac{31}{12} \Rightarrow \frac{28+3}{12} = \frac{31}{12} \Rightarrow \frac{31}{12} = \frac{31}{12}$

7. The sum of two consecutive positive integers is 23. Find the numbers.

Soln:- Let the consecutive integers are $= x, x+1$

Sum of numbers $= x+x+1 = 2x+1$

According to the condition:

$2x+1 = 23$

$\Rightarrow 2x = 23-1 = 22$

$\Rightarrow \frac{2x}{2} = \frac{22}{2} \Rightarrow x = 11$

So 1st number $= x = 11$

2nd number $= x+1 = 11+1 = 12$

8. The sum of three consecutive integers is 159. Find the numbers.

Soln:- Let the required numbers be $= 2x+1, 2x+3, 2x+5$

Sum of numbers $= 2x+1+2x+3+2x+5$

$= 6x+9$

According to the condition:

$6x+9 = 159$

$\Rightarrow 6x = 159-9 = 150$

$\frac{6x}{6} = \frac{150}{6}$ $\div$ing by 6

$\Rightarrow x = 25$

So 1st odd number $= 2x+1 = 2(25)+1 = 50+1 = 51$

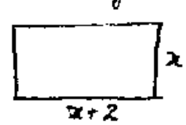
2nd " " $= 2x+3 = 2(25)+3 = 50+3 = 53$

3rd " " $= 2x+5 = 2(25)+5 = 50+5 = 55$

9. The perimeter of a rectangle is 20m. If length of rectangle is 2 meter more than its breadth. Find the length and breadth.

Soln:- Suppose breadth of rectangle $= x$

Length must be $= x+2$



Perimeter of rectangle $= 2(L+B) = 2(x+x+2)$

$= 2(2x+2)$

But according to the condition:

$2(2x+2) = 20m$

$\Rightarrow 2x+2 = 10$ ÷ing by 2

$\Rightarrow 2x = 10-2 = 8$

$\frac{2x}{2} = \frac{8}{2}$ ÷ing b.s by 2

$\Rightarrow x = 4.$

So length of rectangle $= x+2 = 4+2 = 6m$

Breadth of rectangle $= x = 4m.$

10. Age of mother is 13 times the age of her daughter. It will be only 5 times after 4 years. Find their present ages

Soln:: Let age of the daughter $= x$

$\therefore$ age of the mother $= 13x$

After five years their ages $= x+4, 13x+4$

According to the condition:

$13x+4 = 5(x+4)$

$13x+4 = 5x+20$

$\Rightarrow 13x-5x = 20-4$

$\Rightarrow 8x = 16$

dividing both sides by 8 we get

$\frac{8x}{8} = \frac{16}{8} \Rightarrow x = 2$

So age of daughter $= 2$ years

age of mother $= 13x = 13\times2 = 26$ years

11. The sum of two digit 10. If the digits change their places, then the new number is decreased by 18 than the original number. Find the number

Soln:- Let the digit at unit place $= x$
" " " at 10th place $= y$
Then the number is $= x + 10y$.
But the sum the is $10 \Rightarrow x + y = 10$ —— (i)
After changing the position, new number $= y + 10x$
According to the condition;

$$x + 10y = y + 10x + 18$$

∵ dec by 18 So for equality adding 18 to R.H.S

$$\Rightarrow x + 10y - y - 10x = 18$$

$$-9x + 9y = 18$$

÷ing both sides by 9 we get.

$$-x + y = 2 \quad \text{—— (ii)}$$

adding (i) and (ii)

$$x + y = 10$$
$$-x + y = 2$$

$$2y = 12 \Rightarrow \frac{2y}{2} = \frac{12}{2} \Rightarrow \boxed{y = 6}$$

Putting $y = 6$ in eq (i)

$$x + 6 = 10 \Rightarrow x = 10 - 6 = 4$$

So the req number $= x + 10y = 4 + 10(6)$
$= 4 + 60 = 64$

12. Think of a number, if 4 is added to twice of the number, the result comes to 18. Find the number.

Soln:- Let the required number $= x$
Twice the number $= 2x$
adding 4 $= 2x + 4$
According to the condition:

$$2x + 4 = 18$$
$$2x + 4 - 4 = 18 - 4$$
$$2x = 14$$

dividing bs by 2

$$\frac{2x}{2} = \frac{14}{2}$$
$$x = 7$$

So the number is 7

(104)

Radical Equation: The Equation of the type $\sqrt{ax+b}=c$ —(1) is called radical Equation. To solve Eq (1) we will take square on both sides. Then we will verify the value for x. If it is true then this will be the S.S. If false then the S.S will be empty.

Exp 1. Solve $\sqrt{x+5}=3$

Soln: $\sqrt{x+5}=3$

Squaring both sides

$(\sqrt{x+5})^2 = 3^2$

$x+5=9$

$\Rightarrow x+5-5=9-5$

$x=4$

Verification:

$\sqrt{x+5}=3$

Putting $x=4$

$\sqrt{4+5}=3$

$\sqrt{9}=3$

$3=3$

Which is true

So $S.S=\{3\}$

Exp 2 Solve $\sqrt{4x-3}=\sqrt{2x+4}$

Soln: $\sqrt{4x-3}=\sqrt{2x+4}$

Taking square on both sides

$(\sqrt{4x-3})^2=(\sqrt{2x+4})^2$

$4x-3=2x+4$

$4x-2x=4+3$

$2x=7$

$\div$ing b.S by 2

$\frac{2x}{2}=\frac{7}{2}$

$x=7/2$

Verification:

$\sqrt{4x-3}=\sqrt{2x+4}$

Putting $x=7/2$

$\sqrt{4(7/2)-3}=\sqrt{2(7/2)+4}$

$\sqrt{14-3}=\sqrt{7+4}$

$\sqrt{11}=\sqrt{11}$

Which is true.

Hence $S.S=\{7/2\}$

Exp 3 Solve: $\sqrt{3x+2}+6=2$

Soln: $\sqrt{3x+2}+6=2$

$\sqrt{3x+2}+6-6=2-6$

$\sqrt{3x+2}=-4$

Since the sq root of a number can't be negative so $S.S=\{\ \}$

(105)

Ex: 4.2

Solve the following radical Equations

1. $\sqrt{x+5} = 6$

Soln:- $\sqrt{x+5} = 6$

Squaring both Sides we get

$(\sqrt{x+5})^2 = (6)^2$

$x+5 = 36$

$x+5-5 = 36-5$

$x = 31$

Verification:-

Putting $x = 31$ in

$\sqrt{x+5} =$

$\sqrt{31+5} = 6$

$\sqrt{36} = 6$

$6 = 6$

which is true.

Hence S.S = {31}

2. $\sqrt{2x+4} = 6$

Soln:- Taking Square on b.S

$(\sqrt{2x+4})^2 = (6)^2$

$2x+4 = 36$

$2x+4-4 = 36-4$

$2x = 32$

÷ing b.S by 2

$\frac{2x}{2} = \frac{32}{2}$ 16

$x = 16$

Verification:-

Putting $x = 16$ in

$\sqrt{2x+4} = 6$

$\sqrt{2(16)+4} = 6$

$\sqrt{32+4} = 6$

$\sqrt{36} = 6$

$6 = 6$

Which is true

∴ S.S = {16}

3. $\sqrt{x+3} + 2 = 6$

Soln:- $\sqrt{x+3} + 2 = 6$

$\sqrt{x+3} + 2 - 2 = 6 - 2$

$\sqrt{x+3} = 4$

Squaring b.S

$(\sqrt{x+3})^2 = 4^2$

$x+3 = 16$

$x+3-3 = 16-3 \Rightarrow x = 13$

Verification:

Putting $x = 13$ in

$\sqrt{x+3} + 2 = 6$

$\sqrt{13+3} + 2 = 6$

$\sqrt{16} + 2 = 6$

$4 + 2 = 6$

$6 = 6$ True

∴ S.S = {13}

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4. $3\sqrt{x}+7=10$

Soln:- $3\sqrt{x}+7=10$

$3\sqrt{x}+7-7=10-7$

$3\sqrt{x}=3$

$\frac{3\sqrt{x}}{3}=\frac{3}{3}$

$\sqrt{x}=1$

Taking Squaring on b.s

$(\sqrt{x})^2=(1)^2$

$x=1$

Varification:

Putting $x=1$ in

$3\sqrt{x}+7=10$

$3\sqrt{1}+7=10$

$3(1)+7=10$

$3+7=10$

$10=10$

which is true

$\therefore$ S.S $=\{1\}$

5. $\sqrt{4x-1}=\sqrt{3x+1}$

Soln:- $\sqrt{4x-1}=\sqrt{3x+1}$

Squarring both sides

$(\sqrt{4x-1})^2=(\sqrt{3x+1})^2$

$4x-1=3x+1$

$4x-1+1=3x+1+1$

$4x-3x=3x-3x+2$

$x=2$

Varification:

Putting $x=2$ in

$\sqrt{4x-1}=\sqrt{3x+1}$

$\sqrt{4(2)-1}=\sqrt{3(2)+1}$

$\sqrt{8-1}=\sqrt{6+1}$

$\sqrt{7}=\sqrt{7}$

which is true.

$\therefore$ S.S $=\{2\}$

6. $15-\sqrt{x+2}=10$

Soln:- $15-\sqrt{x+2}=10$

$15-15-\sqrt{x+2}=10-15$

$-\sqrt{x+2}=-5$

Squarring both sides

$(-\sqrt{x+2})^2=(-5)^2$

$(-1)^2(\sqrt{x+2})^2=25$

$x+2=25$

$x+2=25$

$\Rightarrow x=25-2=23$

Varification:

$15-\sqrt{x+2}=10$

$15-\sqrt{23+2}=10$

$15-\sqrt{25}=10$

$15-5=10$

$10=10$ True

S.S $=\{23\}$.

(107)

7. $\sqrt{3x-1} + 7 = 4$

Soln:- $\sqrt{3x-1} + 7 = 4$

$\sqrt{3x-1} + 7 - 7 = 4 - 7$

$\sqrt{3x-1} = -3$

Taking square on both sides

$(\sqrt{3x-1})^2 = (-3)^2$

$3x-1 = 9$

$3x-1+1 = 9+1$

$3x = 10$

$\frac{3x}{3} = \frac{10}{3}$

$x = 10/3$

Verification:-

Putting $x = 10/3$ in

$\sqrt{3x-1} + 7 = 4$

$\sqrt{3(10/3)-1} + 7 = 4$

$\sqrt{10-1} + 7 = 4$

$\sqrt{9} + 7 = 4$

$3 + 7 = 4$

$10 = 4$

Which is False

$\therefore$ S.S = { }

8. $\sqrt{4(2x-1)} = 2\sqrt{x+8}$

Soln:- $\sqrt{4(2x-1)} = 2\sqrt{x+8}$

Squaring both sides we get

$(\sqrt{4(2x-1)})^2 = (2\sqrt{x+8})^2$

$4(2x-1) = (2)^2(\sqrt{x+8})^2$

$8x-4 = 4(x+8)$

$8x-4 = 4x+32$

$8x-4+4 = 4x+32+4$

$8x = 4x+36$

$8x-4x = 4x-4x+36$

$4x = 36$

$\frac{4x}{4} = \frac{36}{4} \Rightarrow \boxed{x = 9}$

Verification:

Putting $x = 9$ in

$\sqrt{4(2x-1)} = 2\sqrt{x+8}$

$\sqrt{4(2(9)-1)} = 2\sqrt{9+8}$

$\sqrt{4(18-1)} = 2\sqrt{17}$

$\sqrt{4 \times 17} = 2\sqrt{17}$

$\sqrt{4} \cdot \sqrt{17} = 2\sqrt{17}$

$2\sqrt{17} = 2\sqrt{17}$

which is true

So S.S = {9}

Absolute Value:- Let x be any number then absolute value of x denoted by $|x|$ and defined as $|x| = x$ if $x > 0$

$|x| = 0$ if $x = 0$

$|x| = -x$ if $x < 0$

e.g $|5| = 5$, $|-5| = -(-5) = 5$.

i.e $|x|$ is a function which converts the $-ve$ value to positive.

Properties of absolute value (i) $|ab| = |a||b|$

(ii) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ (iii) $|a+b| \leq |a| + |b|$

Note If a and b have same sign then $|a+b| = |a| + |b|$ and if a and b have opposite sign then $|a+b| < |a| + |b|$

Ex # 4.3

Solve the following Equations:

1. $|4x| = 8$

Soln:- There are two possibilities

$4x = 8$ and $4x = -8$

Dividing by 4

$\frac{4x}{4} = \frac{8}{4}$ and $\frac{4x}{4} = \frac{-8}{4}$

$x = 2$ and $x = -2$.

$\therefore$ S.S $= \{\pm 2\} = \{2, -2\}$

2. $|x-1| = 7$

Soln:- $|x-1| = 7$

There are two possibilities

$x-1 = 7$ and $x-1 = -7$

$x-1+1 = 7+1$ | $x-1+1 = -7+1$

$x = 8$ | $x = -6$

$\therefore S.S = \{8, -6\}$.

3. $\left|\frac{x}{5}\right| = 6$

Sol:- There are two possibilities

$\frac{x}{5} = 6$ —(i) and $\frac{x}{5} = -6$ —(ii)

multiplying (i) and (ii) by 5

$5 \cdot \frac{x}{5} = 6 \times 5$ and $\frac{x}{5} \times 5 = -6 \times 5$

$x = 30$ and $x = -30$

$\therefore S.S = \{30, -30\}$

4. $|15x-10| = 35$

Soln:- There are two possibilities

$15x-10 = 35$ —(i) and $15x-10 = -35$ —(ii)

adding 10 to both sides of Eq (i) and (ii)

$15x-10+10 = 35+10$ and $15x-10+10 = -35+10$

$15x = 45$ and $15x = -20$

$\frac{15x}{15} = \frac{45}{15}$ $\qquad$ $\frac{15x}{15} = \frac{-20}{15}$

$x = 3$ $\qquad$ $x = -4/3$

$S.S = \{3, -4/3\}$

(110)

5. $\left|\frac{x+5}{3}\right| = \frac{7}{2}$

Soln:- There are two possibilities i.e

$\frac{x+5}{3} = \frac{7}{2}$ —(i) and $\frac{x+5}{3} = \frac{-7}{2}$ —(ii)

Using cross multiplication

$2(x+5) = 3\times 7$

$2x+10 = 21$

$2x+10-10 = 21-10$

$2x = 11$

$\frac{2x}{2} = \frac{11}{2}$

$x = 11/2$

By cross multiplication

$2(x+5) = 3\times(-7)$

$2x+10 = -21$

$2x+10-10 = -21-10$

$2x = -31$

$\frac{2x}{2} = -\frac{31}{2}$

$x = -\frac{31}{2}$

$S.S = \left\{\frac{11}{2}, -\frac{31}{2}\right\}$

6. $|5x-13| + 2 = 14$

Soln:- $|5x-13| + 2 - 2 = 14 - 2$

$|5x-13| = 12$

Now there are two possibilities

$5x-13 = 12$ —(i) and $5x-13 = -12$ —(ii)

adding 13 to both sides of eq(i) and (ii)

$15x-13+13 = 12+13$ and $15x-13+13 = -12+13$

$15x = 25$

$\frac{15x}{15} = \frac{25}{15}$

$x = \frac{5}{3}$

$15x = 1$

$\frac{15x}{15} = \frac{1}{15}$

$x = \frac{1}{15}$

$\therefore S.S = \left\{5/3, \frac{1}{15}\right\}$

(iii)

7. $|4x-1| = 7$

Soln:- $|4x-1| = 7$

There are two possibilities:

$4x-1=7$ —(i) and $4x-1=-7$ —(ii)

adding 1 to both sides of (i) and (ii) we get

$4x-1+1=7+1$ and $4x-1+1=-7+1$

$4x=8$	$4x=-6$
$\frac{4x}{4}=\frac{8}{4}$	$\frac{4x}{4}=-\frac{6}{4}$
$x=2$	$x=-\frac{3}{2}$

$\therefore$ S.S $= \{2, -\frac{3}{2}\}$

8. $|9x+11| - \frac{2}{5} = \frac{8}{5}$

Soln:- $|9x+11| - \frac{2}{5} = \frac{8}{5}$

$|9x+11| - \frac{2}{5} + \frac{2}{5} = \frac{8}{5} + \frac{2}{5}$

$|9x+11| = \frac{8+2}{5} = \frac{10}{5} = 2$

$\Rightarrow |9x+11| = 2$

Now there are two possibilities i.e

$9x+11=2$ —(i) and $9x+11=-2$ —(ii)

$9x+11-11=2-11$	$9x+11-11=-2-11$
$9x=-9$	$9x=-13$
$\frac{9x}{9}=-\frac{9}{9}$	$\frac{9x}{9}=-\frac{13}{9}$
$x=-1$	$x=\frac{-13}{9}$

S.S $= \{-1, \frac{-13}{9}\}$

Linear Inequalities :- The expression of the type $ax \geq b$ or $ax \leq b$ or $ax > b$ or $ax < b$ is called linear inequality.

(i) $a \geq b$ means that a is greater or equal to b

(ii) $a > b$ means that a is greater than b

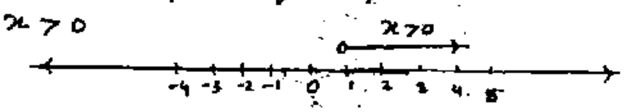
(iii) $a \leq b$ means that a is less or equal to b

(iv) $a < b$ means that a is less than b

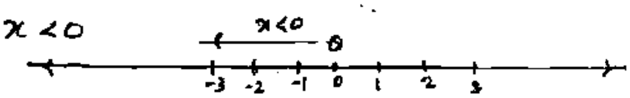
EX : 4.4

Depict the following inequalities on a real line

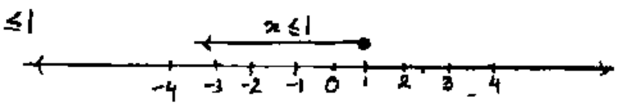
(1) $x > 0$



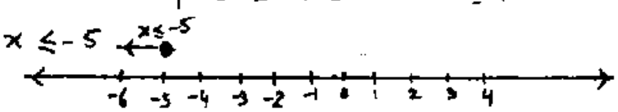
(2) $x < 0$



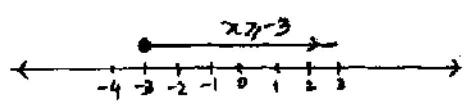
3. $x \leq 1$



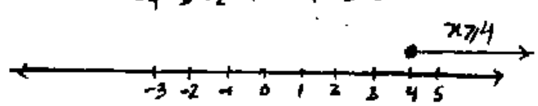
4. $x \leq -5$



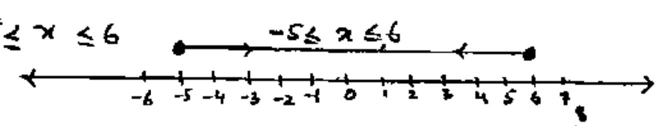
5. $x \geq -3$



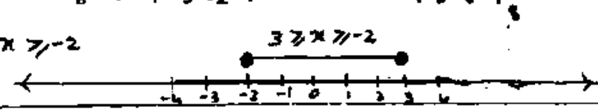
6. $x \geq 4$



7. $-5 \leq x \leq 6$

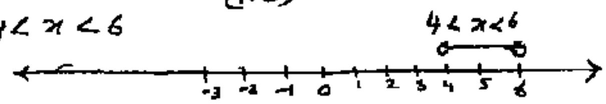


8. $3 \geq x \geq -2$

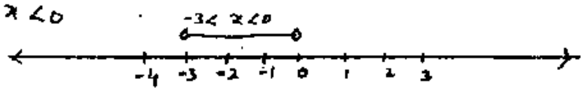


(113)

9) $4 < x < 6$



10. $-3 < x < 0$



EX: 4.5

Find the solution set of the following inequalities:

1. $5x+4 < 34 \quad x \in N = \{1,2,3,\cdots\}$

Soln:- $5x+4 < 34$

$5x+4-4 < 34-4$

$5x < 30$

$\frac{5x}{5} < \frac{30}{5}$

$x < 6$

$\therefore S.S = \{1,2,3,4,5\}$

2. $3x-5 > 1 \quad x \in N = \{1,2,3,\cdots\}$

Soln:- $3x-5+5 > 1+5$

$3x > 6$

$\frac{3x}{3} > \frac{6}{3}$

$x > 2 \quad \therefore S.S = \{3,4,5,\cdots\}$

3. $3(x+4) > x+5 \quad x \in R$

Soln $3(x+4) > x+5$

$3x+12 > x+5$

$3x-x > 5-12$

$2x > -7 \qquad \therefore S.S = \{x \in R : x > -7/2\}$

$\frac{2x}{2} > -7/2$

$x > -7/2$

(114).

4. $3x-4>2 \quad x \in \{1,2,3,4,5,6\}$

Soln: $3x-4>2$

$3x+4-4>2+4$ ∵ adding 4 on b.s

$3x>6$

dividing both sides by 3

$\frac{3x}{3}>\frac{6}{3}$

$\Rightarrow x>2$

So the $S.S=\{3,4,5,6\}$

5. $2(x-3)\geq x-2 \quad , x\in\{-2,-1,0,1,2,3,4\}$

Soln: $2(x-3)\geq x-2$

$2x-6\geq x-2$ by adding 6 on b.s

$2x-6+6\geq x-2+6$

$2x\geq x+4$ by subtracting x

$2x-x\geq x-x+4$

$x\geq 4$

∴ $S.S=\{4\}$ ∵ There is only 4 in the given set

6. $\frac{2x+6}{2}\geq\frac{x-9}{3} \quad x\in R.$

Soln: Multiplying both sides by 6

$3\,6\left(\frac{2x+6}{2}\right)\geq 6^2\left(\frac{x-9}{3}\right)$

$3(2x+6)\geq 2(x-9)$

$6x+18\geq 2x-18$ by subtracting +18 on b.s

$6x+18-18\geq 2x-18-18$

$6x\geq 2x-36$

$6x-2x\geq 2x-2x-36$

$4x\geq -36 \Rightarrow \frac{4x}{4}\geq\frac{-36}{4}\Rightarrow x\geq -9$

∵ $S.S=\{x\in R: x\geq -9\}$

7. $\frac{x-1}{3} \leq \frac{1-x}{2}$

Soln:- Multiplying both sides by 6 we get.

$6\left(\frac{x-1}{3}\right) \leq 6\left(\frac{1-x}{2}\right)$

$2(x-1) \leq 3(1-x)$

$2x-2 \leq 3-3x$

$2x-2+2 \leq 3+2-3x$ by adding 2 on b.s

$2x \leq 5-3x$ ∴ adding 3x.

$2x+3x \leq 5-3x+3x$

$5x \leq 5$

$\frac{5x}{5} \leq \frac{5}{5} \Rightarrow x \leq 1$

$\therefore S.S = \{x \in \mathbb{Z} : x \leq 1\} = \{\ldots, -2, -1, 0\}$

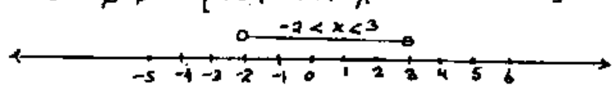
Find the Solution Sets of the inequalities and depict them on number line.

8. $-2 < x < 3$

Soln :- The inequality can be written as

$-2 < x$ and $x < 3$

$\therefore S.S = \{x \mid x \in R \wedge -2 < x < 3\}$

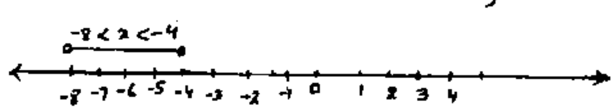


9. $-4 > x > -8$

Soln:- The inequality can be written as

$-4 > x$ and $x > -8$

$\therefore S.S = \{x \mid x \in R \wedge -8 < x < -4\}$



(116)

10. $1 < x+4 \leq 4$

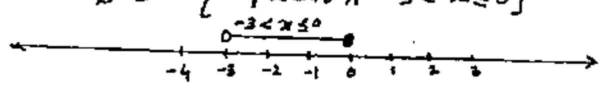
Soln: The inequality can be written into two parts as

$1 < x+4$ and $x+4 \leq 4$

$1-4 < x+4-4$ | $x+4-4 \leq 4-4$

$-3 < x$ | $x \leq 0$

$\therefore$ S.S = $\{x \mid x \in \mathbb{R} \wedge -3 < x \leq 0\}$



11. $-5 \geq x \geq -2$

Soln: The inequality can be written as

$-5 \geq x$ and $x \geq -2$

or $x \leq -5$ and $-2 \leq x$

As it is impossible that $x \leq -5$ and $x \geq -2$

so its S.S does not exists

12. $5 \leq x+2 \leq 8$

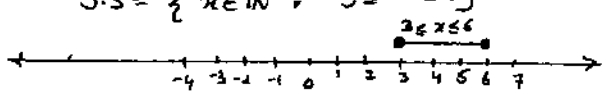
Soln: $5 \leq x+2 \leq 8$

Subtraction 2 from the inequality we get

$5-2 \leq x+2-2 \leq 8-2$

$3 \leq x \leq 6$

S.S = $\{x \in \mathbb{R} : 3 \leq x \leq 6\}$



Ex # 4.6

1. Read the following and encircle 'T' for True and 'F' for false

(i) $\frac{x}{4} = 3$ is Linear Equation in on varible (T)

(ii) The S.S of $5x+3=8$ is $\{1\}$ (T)

(117)

(iii) The S.S of $\sqrt{x}+2=4$ is $\{2\}$. (F)

(iv) $|x| = -x$ if $x > 0$. (F)

(v) if $x<y$ and $y<z$ Then $x>z$ (F)

2. Fill in the blanks:

(i) If $x<y \Rightarrow xz$ __<__ yz for $z>0$.

(ii) '≤' means __Less__ or equal to.

(iii) The Solution Set of $|x|-2=0$ is __$\{\pm 2\}$__.

(iv) The S.S for $\sqrt{x+5}=3$ is __$\{4\}$__

(v) The S.S of $0 < x+2 < 3$ when $x \in \mathbb{Z}$ is __$\{-1, 0\}$__.

3. Write the correct option against The given box.

(i) b (ii) a (iii) d (iv) b

(v) c (vi) c (vii) d (viii) b (ix) c

(x) c (xi) d (xii) a (xiii) b (xiv) b

REVIEW EXERCISE 4

1) Find the Solution sets of The following Eqs and verify

(i) $\frac{5}{2x-3} = \frac{4}{3x-1}$

Sol:- By Cross multiplication

$5(3x-1) = 4(2x-3)$

$15x-5 = 8x-12$

$\Rightarrow 15x = 8x-12+5$

$15x-8x = -7$

$7x = -7$

$\Rightarrow \frac{7x}{7} = -\frac{7}{7}$

$x = -1$

Verification:

Putting $x=-1$ in

$\frac{5}{2x-3} = \frac{4}{3x-1}$

$\frac{5}{2(-1)-3} = \frac{4}{3(-1)-1}$

$\frac{5}{-2-3} = \frac{4}{-3-1}$

$\frac{5}{-5} = \frac{4}{-4}$

$-1 = -1$ True

$\therefore S.S = \{-1\}$

(118)

(ii) $\frac{3}{x+1} = \frac{-3}{7}$

Soln:- By using cross multiplication

$-3(x+1) = 3\times 7$

$-3x-3 = 21$

$\Rightarrow -3x = 21+3$

$\Rightarrow -3x = 24$

$\Rightarrow \frac{-3x}{-3} = \frac{24}{-3}$

$\Rightarrow \boxed{x = -8}$

Verification:

Putting $x=-8$ in

$\frac{3}{x+1} = \frac{-3}{7}$

$\frac{3}{-8+1} = \frac{-3}{7}$

$\frac{3}{-7} = \frac{-3}{7}$

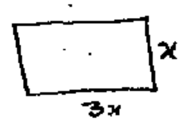
which is true

$\therefore$ S.S $= \{-8\}$

2. Perimeter of a rectangular field is 48 m. Its length is three times its width. Find the area of rectangular field.

Soln:- Let breadth $= x$ m

then length $= 3x$



Perimeter $= 2(L+W) = 2(x+3x) = 2(4x) = 8x$

But Perimeter $= 48$ m (Given)

$\therefore 8x = 48$ m

$\div$ ing both sides by 8.

$\frac{8x}{8} = \frac{48}{8} \Rightarrow \boxed{x = 6m}$

So Width $= x = 6$ m, Length $= 3x = 3\times 6 = 18$ m

$\therefore$ The required area $= L\times W = 6\times 18 = 108\ m^2$

Solve the following Equations.

3. $\sqrt{2x-3} + 5 = 10$

Soln:- $\sqrt{2x-3} + 5 = 10$

$\sqrt{2x-3} + 5 - 5 = 10 - 5$

(119)

$\sqrt{2x-3} = 5$

Squarring both sides.

$(\sqrt{2x-3})^2 = (5)^2$

$2x-3 = 25$

$2x-3+3 = 25+3$

$2x = 28$

$\Rightarrow \frac{2x}{2} = \frac{28}{2}$

$x = 14$

Verification: Putting $x = 14$ in

$\sqrt{2x-3}+5 = 10$

$\sqrt{2(14)-3}+5 = 10$

$\sqrt{28-3}+5 = 10$

$\sqrt{25}+5 = 10$

$5+5 = 10$

$10 = 10$

Which is true

$\therefore S.S = \{14\}$

4. $5\sqrt{x}+7 = 2$

Soln: $5\sqrt{x}+7 = 2$

$5\sqrt{x}+7-7 = 2-7$

$5\sqrt{x} = -5$

$\frac{5\sqrt{x}}{5} = \frac{-5}{5}$

$\Rightarrow \sqrt{x} = -1$

Squarring both sides

$(\sqrt{x})^2 = (-1)^2$

$x = 1$.

Verification: Putting $x=1$ in

$5\sqrt{x}+7 = 2$

$5\sqrt{1}+7 = 2$

$5(1)+7 = 2$

$12 = 2$

Which is false as $12 \neq 2$

$\therefore S.S = \{\ \}$.

5. $|3x-4| - \frac{3}{4} = 2$

Soln: $|3x-4| - \frac{3}{4} = 2$

$|3x-4| = \frac{2}{1}+\frac{3}{4}$

$|3x-4| = \frac{8+3}{4} = \frac{11}{4}$ —— ①

There are two possibilities for Eq ①

(120)

$3x-4 = \frac{11}{4}$ and $3x-4 = -\frac{11}{4}$

$3x-4+4 = \frac{11}{4}+\frac{4}{1}$ | $3x-4+4 = -\frac{11}{4}+\frac{4}{1}$

$3x = \frac{11+16}{4}$ | $3x = \frac{-11+16}{4}$

$3x = \frac{27}{4}$ | $3x = \frac{5}{4}$

$\frac{1}{3}\times 3x = \frac{27}{4}\times \frac{1}{3}$ | $\frac{1}{3}\times 3x = \frac{5}{4}\times\frac{1}{3}$

$x = 9/4$ | $x = \frac{5}{12}$

$\therefore S.S = \{9/4, \frac{5}{12}\}$

6. $\left|\frac{3x+2}{2}\right| + 2 = 12$

Soln: $\left|\frac{3x+2}{2}\right| + 2 = 12$

$\left|\frac{3x+2}{2}\right| + 2 - 2 = 12 - 2$

$\left|\frac{3x+2}{2}\right| = 10$ —— (i)

Now there are two possibilities for Equation (i)

$\frac{3x+2}{2} = 10$ and $\frac{3x+2}{2} = -10$

×ing both sides by 2.

$2\times\frac{3x+2}{2} = 2\times 10$ | $2\times\frac{3x+2}{2} = 2\times(-10)$

$3x+2 = 20$ | $3x+2 = -20$

$3x+2-2 = 20-2$ | $\Rightarrow 3x+2-2 = -20-2$

$3x = 18$ | $3x = -22$

$\frac{3x}{3} = \frac{18}{3}$ | $\frac{3x}{3} = \frac{-22}{3}$

$\Rightarrow x = 6$ | $x = \frac{-22}{3}$

$S.S = \{6, -\frac{22}{3}\}$

(121)

7. $\left|\frac{x+1}{4}\right| = 5$

Soln:- There are two possibilities.

$\frac{x+1}{4} = 5$ and $\frac{x+1}{4} = -5$

x ing these equations by 4 we get

$4 \times \frac{x+1}{4} = 4 \times 5$ and $4 \times \frac{x+1}{4} = 4 \times (-5)$

$x+1 = 20$ and $x+1 = -20$

$\Rightarrow x = 20-1$ $\Rightarrow x = -20-1$

$\Rightarrow x = 19$ and $x = -21$

$\therefore S.S = \{19, -21\}$

8. $4(2x-3) \geq 3(x+1)$, $x \in N$

$\Rightarrow 8x - 12 \geq 3x + 3$

$8x - 12 + 12 \geq 3x + 3 + 12$ by adding 12 on b.s

$8x \geq 3x + 15$

$8x - 3x \geq 3x - 3x + 15$ by subtracting 3x

$5x \geq 15$

$\frac{5x}{5} \geq \frac{15}{5}$ ... diving b.s by 5

$\Rightarrow x \geq 3$ and $x \in N = \{1, 2, 3, 4, \cdots\}$

$\therefore S.S = \{3, 4, 5, 6, \cdots\}$

9. $-4 < 3x+5 < 10$, $x \in R$

Soln:- $-4 < 3x+5 < 10$

Subtracting 5 from both sides of the inequality

$-4-5 < 3x+5-5 < 10-5$

$-9 < 3x < 5 \Rightarrow \frac{-9}{3} < \frac{3x}{3} < 5/3$

$\Rightarrow -3 < x < 5/3$ $\therefore S.S = \{x \in R \wedge -3 < x < 5/3\}$

Unit 5

QUADRATIC EQUATIONS

Quadratic Equation :- The equation of the type $ax^2+bx+c=0$ where $a \neq 0$ and all $a,b,c \in R$ is called quadratic equation. e.g $2x^2+4x-3=0$

Notes:- (i) If in the Quadratic equation $a=0$, the Eq reduces to $bx+c$ which is Linear.

(ii) The solution set of $ax^2+bx+c=0$ contains two values

(iii) If $ab=0$ then either $a=0$ or $b=0$ or both are zero

Quadratic Equations can be solved by three methods

(i) By factorization method
(ii) By Completing Square method
(iii) By quadratic formula.

Ex: 5.1

1. Which of the following are quadratic Equations.

(i) $x^2-\frac{1}{x}+3=0$ (This is not quadratic Eq)

(ii) $x^4-2x+1=0$ (This is not quadratic Eq)

(iii) $x-\frac{6}{x}+5=0$ (This is quadratic Eq)

(iv) $x(2x^2+4)$ (This is not quadratic Eq)

(v) $\frac{1}{x^2}=1$ (This is quadratic Eq)

(vi) $x=0$ (This is not quadratic Eq)

(123)

2. Arrange the following equations in Standard form $ax^2+bx+c=0$

(i) $x^2=3x+1$

Soln:- $x^2=3x+1$

$\Rightarrow x^2-3x-1=0$

(ii) $\frac{1}{x^2}-2+\frac{3}{x}=0$

Soln: $\frac{1}{x^2}-2+\frac{3}{x}=0$

$x^2\cdot\frac{1}{x^2}-x^2\cdot 2+x^2\cdot\frac{3}{x}=0\cdot x^2$

$1-2x^2+3x=0$

$\Rightarrow 2x^2-3x-1=0$

(iii) $x^2-3=0$

$\Rightarrow x^2-0\cdot x-3=0$

(iv) $x^2=5x-1$

$\Rightarrow x^2-5x+1=0$

(v) $3x^2-x=0$

$\Rightarrow 3x^2-x+0=0$

Solution of quadratic Equations:

Expl#1 Solve $2x^2+5x+3=0$ by factorization

Soln:- $2x^2+5x+3=0$

$2\times3=6$
2×3
$2+3=5$

$\Rightarrow 2x^2+(2+3)x+3=0$

$2x^2+2x+3x+3=0$

$2x(x+1)+3(x+1)=0$

$(x+1)(2x+3)=0$

$\Rightarrow x+1=0$ or $2x+3=0$

$\Rightarrow x=-1$ or $2x=-3$

$\frac{2x}{2}=\frac{-3}{2}$

$x=-3/2$

$S.S=\{-1,-3/2\}$

Exp 3 Solve by factorization: $x^2 - 5x + 4 = 0$

Soln:- $x^2 - 5x + 4 = 0$

$x^2 - (4+1)x + 4 = 0$

$x^2 - 4x - 1\cdot x + 4 = 0$

$x(x-4) - 1(x-4) = 0$

$(x-1)(x-4) = 0$

$\Rightarrow x - 1 = 0$ or $x - 4 = 0$

$\Rightarrow x = 1$ or $x = 4$

$S.S = \{1, 4\}$

$1 \times 4 = 4$
-1×-4
$(-1) + (-4) = -5$

Solution of quadratic Equation by Completing Square

Suppose we want to find the soln of $ax^2 + bx + c = 0$ by completing square, then the following steps will be carried out

(i) Convert the constant term to R.H.S i.e $ax^2 + bx = -c$

(ii) $\div$ing both sides by a i.e $x^2 + \frac{b}{a}x = -\frac{c}{a}$

(iii) Adding the square of half of the coefficent of x to both sides i.e

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

(iv) Take the square root on both sides.

Exp 1: Solve by completing method: $2x^2 - 6x + 4 = 0$

Soln:- $2x^2 - 6x + 4 = 0$

$\Rightarrow 2x^2 - 6x = -4$

$\frac{2x^2}{2} - \frac{6x}{2} = \frac{-4}{2}$

$\Rightarrow x^2 - 3x = -2$ —— (1)

adding $(3/2)^2$ to both sides of (1)

$x^2 - 3x + (3/2)^2 = -2 + (3/2)^2$

$\Rightarrow (x - 3/2)^2 = -\frac{2}{1} + \frac{9}{4} = \frac{-8+9}{4} = \frac{1}{4}$

Taking the square-roote on both sides

$\sqrt{(x-\frac{3}{2})^2} = \sqrt{\frac{1}{4}} \Rightarrow x - \frac{3}{2} = \pm\frac{1}{2}$

$\Rightarrow x - \frac{3}{2} = \frac{1}{2}$ and $x - \frac{3}{2} = -\frac{1}{2}$

$x = \frac{1}{2} + \frac{3}{2}$ | $x = -\frac{1}{2} + \frac{3}{2}$

$x = \frac{1+3}{2} = \frac{4}{2} = 2$ | $x = \frac{-1+3}{2} = \frac{2}{2} = 1$

$S.S = \{2, 1\}$

Ex: 5.2

1. Solve The following quadratic Eq by factorization method

(i) $1 + \frac{15}{x^2} = \frac{16}{x}$

Soln:- Multiplying the Eq by x^2

$1 \cdot x^2 + \frac{15}{x^2} \cdot x^2 = \frac{16}{x} x^2$

$x^2 + 15 = 16x \Rightarrow x^2 - 16x + 15 = 0$

$\Rightarrow x^2 - 15x - x + 15 = 0$

$x(x-15) - 1(x-15) = 0$

$(x-1)(x-15) = 0$

$\Rightarrow x - 1 = 0$ or $x - 15 = 0$

$x = 1$ or $x = 15$

$S.S = \{1, 15\}$

(126)

(ii) $x(3x-5) = (x-6)(x-7)$

Soln:- $x(3x-5) = (x-6)(x-7)$

$\Rightarrow 3x^2 - 5x = x^2 - 7x - 6x + 42$

$\Rightarrow 3x^2 - 5x = x^2 - 13x + 42$

$\Rightarrow 3x^2 - x^2 - 5x + 13x - 42 = 0$

$\Rightarrow 2x^2 + 8x - 42 = 0$

÷ing both sides by 2

$\frac{2x^2}{2} + \frac{8x}{2} - \frac{42}{2} = 0 \Rightarrow x^2 + 4x - 21 = 0$

$\Rightarrow x^2 + 4x - 21 = 0$

$\Rightarrow x^2 + 7x - 3x - 21 = 0$

$x(x+7) - 3(x+7) = 0$

$\Rightarrow (x+7)(x-3) = 0$

$\Rightarrow x+7 = 0$ or $x-3 = 0$

$\Rightarrow x = -7$ or $x = 3$

$S.S = \{-7, 3\}$

-21
$\wedge$
$7x - 3$

(iii) $\frac{x}{x+6} + \frac{3}{x+4} = \frac{6}{(x+6)(x+4)}$

Soln:- $\frac{x(x+4) + 3(x+6)}{(x+6)(x+4)} = \frac{6}{(x+6)(x+4)}$

$\Rightarrow \frac{x^2 + 4x + 3x + 18}{(x+6)(x+4)} = \frac{6}{(x+6)(x+4)}$

$x^2 + 7x + 18 - 6 = 0$

$\Rightarrow x^2 + 7x + 12 = 0$

$\Rightarrow x^2 + 3x + 4x + 12 = 0$

$x(x+3) + 4(x+3) = 0$

$(x+3)(x+4) = 0$

$\Rightarrow x+3 = 0$ or $x+4 = 0$

$\Rightarrow x = -3,\ x = -4$

So $S.S = \{-3, -4\}$

(127)

(iv) $6x + \frac{1}{x} = 7$

Soln:- multiplying both sides by x.

$6x(x) + x\left(\frac{1}{x}\right) = 7x \Rightarrow 6x^2 + 1 = 7x = 0$

$\Rightarrow 6x^2 - 7x + 1 = 0$

$\Rightarrow 6x^2 - 6x - x + 1 = 0$

$6x(x-1) - 1(x-1) = 0$

$(x-1)(6x-1) = 0$

$\Rightarrow x - 1 = 0$ or $6x - 1 = 0$

$\Rightarrow x = 1$ or $\frac{6x}{6} = \frac{1}{6}$

$x = 1$, or $x = 1/6$

$S.S = \{1, 1/6\}$

6
$-6x \cdot -1$

(v) $64x^2 + 16x = -1$

Soln:- $64x^2 + 16x = -1 \Rightarrow 64x^2 + 16x + 1 = 0$

$64x^2 + 8x + 8x + 1 = 0$

$8x(8x+1) + 1(8x+1) = 0$

$(8x+1)(8x+1) = 0 \Rightarrow \sqrt{(8x+1)^2} = \sqrt{0^2}$

$\Rightarrow 8x + 1 = 0 \Rightarrow 8x = -1 \Rightarrow \boxed{x = -\frac{1}{8}}$

$S.S = \{-\frac{1}{8}\}$

64
8×8 and $8 + 8 = 16$

2. Solve the following quadratic Equations by Comp. Squa

(i) $x^2 - 2x - 444 = 0$

Soln:- $x^2 - 2x = 444$

$x^2 - 2x + (1)^2 = 444 + (1)^2$ adding $(1)^2$

$(x)^2 - 2(x)(1) + (1)^2 = 445$

$(x-1)^2 = 445$

(128)

Taking Square root on both sides.

$\sqrt{(x-1)^2} = \sqrt{445}$

$x-1 = \pm\sqrt{445}$

$\Rightarrow x = 1 \pm \sqrt{445}$

$\therefore$ S.S $= \{1 \pm \sqrt{445}\}$

(ii) $5x^2 - 10x - 890 = 0$

Soln: $5x^2 - 10x = 890$

$\frac{5x^2}{5} - \frac{10x}{5} = \frac{890}{5}$ dividing by 5

$x^2 - 2x = 178$

Adding the Square of half of the coeff of x i.e $\frac{2}{2} = 1$

$x^2 - 2x + (1)^2 = 178 + 1^2$

$\Rightarrow (x)^2 - 2(x)(1) + (1)^2 = 179$

$(x-1)^2 = 179$

Taking Square root on both sides

$\sqrt{(x-1)^2} = \sqrt{179} \Rightarrow x-1 = \pm\sqrt{179}$

$\Rightarrow x = 1 \pm \sqrt{179}$

S.S $= \{1 \pm \sqrt{179}\}$

(iii) $x^2 - 4x - 921 = 0$

Soln: $x^2 - 4x = 921$

Coeff of $x = 4$

half of coeff of $x = \frac{4}{2} = 2$

adding $(2)^2$ on both sides

$x^2 - 4x + (2)^2 = 921 + (2)^2$

$(x)^2 - 2(2)(x) + (2)^2 = 921 + 4 = 925$

(124)

$\Rightarrow (x-2)^2 = 925$

$\Rightarrow \sqrt{(x-2)^2} = \sqrt{925}$

5	925
5	185
	37

$x-2 = \pm\sqrt{5\times5\times37} = \pm 5\sqrt{37}$

$\Rightarrow x = 2 \pm 5\sqrt{37}$

$S.S = \{2 \pm 5\sqrt{37}\}$

(iv) $x^2 = 25x - 24$

$\Rightarrow x^2 - 25x = -24$

Coeff of $x = 25$
Half of it $= \frac{25}{2}$

adding $\left(\frac{25}{2}\right)^2$ on both sides we get

$x^2 - 25x + \left(\frac{25}{2}\right)^2 = -24 + \left(\frac{25}{2}\right)^2$

$(x)^2 - 2\left(\frac{25}{2}\right)(x) + \left(\frac{25}{2}\right)^2 = \frac{-24}{1} + \frac{625}{4} = \frac{-96+625}{4}$

$\Rightarrow \left(x - \frac{25}{2}\right)^2 = \frac{529}{4}$

$\Rightarrow \sqrt{\left(x - \frac{25}{2}\right)^2} = \sqrt{\frac{529}{4}} = \pm\frac{23}{2}$

$\because \sqrt{529} = \sqrt{23^2} = 23$

$x - \frac{25}{2} = \pm\frac{23}{2}$

$\Rightarrow x - \frac{25}{2} = \frac{23}{2}$ or $x - \frac{25}{2} = -\frac{23}{2}$

$\Rightarrow x = \frac{23}{2} + \frac{25}{2}$ $\quad x = -\frac{23}{2} + \frac{25}{2}$

$x = \frac{48}{2} = 24$ $\quad x = \frac{-23+25}{2} = \frac{2}{2} = 1$

$S.S = \{24, 1\}$

(v) $x^2 - 10x + 10 = 0$

Soln:- $x^2 - 10x = -10$

Coeff of $x = -10$
Half of it $= -5$

adding $(5)^2$ on b. sides

$x^2 - 10x + (5)^2 = -10 + (5)^2$

(130)

$(x)^2 - 5(x)(2) + (5)^2 = -10 + 25$

$(x-5)^2 = 15$

Taking Square root on both sides

$\sqrt{(x-5)^2} = \sqrt{15} \Rightarrow x-5 = \pm\sqrt{15}$

$\Rightarrow x = 5 \pm \sqrt{15}$

$S.S = \{5 \pm \sqrt{15}\}$

$2x^2 = 32x + 34.$

$\Rightarrow 2x^2 - 32x = 34$

$\div$ ing by 2 we get

$x^2 - 16x = 17$

Coeff of $x = 16$

Half of it $= \frac{16}{2} = 8$

adding 8^2 on both sides we get

$x^2 - 16x + 8^2 = 17 + 8^2$

$(x)^2 - 2(x)(8) + 8^2 = 17 + 64 = 81$

$\Rightarrow (x-8)^2 = 81$

$\sqrt{(x-8)^2} = \sqrt{81} \Rightarrow x - 8 = \pm 9$

$\Rightarrow x - 8 = 9$ or $x - 8 = -9$

$x = 9 + 8$ or $x = -9 + 8$

$x = 17$ or $x = -1$

$S.S = \{17, -1\}$

QUADRATIC FORMULA:- If a quadratic Eq $ax^2 + bx + c = 0$ can,t be solved easily by factorizatio or Completing square then Quadratic formula is used and given by: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Derivation of Quadratic formula:- The formula can be derived by solving $ax^2+bx+c=0$ using completing square.

Now;

$$ax^2+bx+c=0$$

$$\Rightarrow ax^2+bx=-c$$

$$\frac{ax^2}{a}+\frac{bx}{a}=-\frac{c}{a}$$

$$\Rightarrow x^2+\frac{b}{a}x=-\frac{c}{a}$$

Coefficient of $x=\frac{b}{a}$

Half of it $=\frac{b}{2a}$

adding $\left(\frac{b}{2a}\right)^2$ on b.s we get

$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=-\frac{c}{a}+\left(\frac{b}{2a}\right)^2$$

$$\Rightarrow \left(x+\frac{b}{2a}\right)^2=-\frac{c}{a}+\frac{b^2}{4a^2}=\frac{-4ac+b^2}{4a^2}$$

$$\Rightarrow \sqrt{\left(x+\frac{b}{2a}\right)^2}=\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$\Rightarrow x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Hence $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Ex # 5.3

1. Solve the following Equations by Quadratic formula.

$x^2-8x+15=0$

Here $a=1$, $b=-8$, $c=15$

Quadratic formula: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ ——(i)

Putting the values of a, b and c in Eq.(i)

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)} = \frac{8 \pm \sqrt{64-60}}{2}$$

$$x = \frac{8 \pm \sqrt{4}}{2} = \frac{8 \pm 2}{2}$$

$$\Rightarrow x = \frac{8+2}{2} \quad \text{or} \quad x = \frac{8-2}{2}$$

$$x = \frac{10}{2} = 5 \quad \text{or} \quad x = \frac{6}{2} = 3$$

$$S.S = \{5, 3\}$$

2. $3x^2 - x = 2$

Soln.: $3x^2 - x - 2 = 0$

Here $a = 3$, $b = -1$, $c = -2$

Quadratic formula : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ —(i)

Putting the values of a, b & c in Eq.(i)

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-2)}}{2 \times 3} = \frac{1 \pm \sqrt{1+24}}{6}$$

$$= \frac{1 \pm \sqrt{25}}{6} = \frac{1 \pm 5}{6}$$

$$\Rightarrow x = \frac{1+5}{6} \quad \text{or} \quad x = \frac{1-5}{6}$$

$$= \frac{6}{6} = 1 \quad \text{or} \quad x = \frac{-4}{6} = -\frac{2}{3}$$

$$\therefore S.S = \{1, -2/3\}$$

3. $4x^2 + 3x = 0$

Soln.: Here $a = 4$, $b = 3$, $c = 0$

Quadratic formula : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ —(i)

(133)

Putting the values of a, b and c in Equation

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(4)(0)}}{2 \times 4} = \frac{-3 \pm \sqrt{9-0}}{8} = \frac{-3 \pm 3}{8}$$

$$\Rightarrow \quad x = \frac{-3+3}{8} \quad \text{or} \quad x = \frac{-3-3}{8}$$

$$x = \frac{0}{8} = 0 \quad \text{or} \quad x = -\frac{6}{8} = -\frac{3}{4}$$

$$S.S = \{0, -3/4\}$$

4). $3x(x-2)+1=0$

Soln: $3x(x-2)+1=0 \Rightarrow 3x^2-6x+1=0$

Here; $a=3,\ b=-6,\ c=1$

Quadratic formula is: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ ——(i)

Putting the above values in Equation (i)

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(1)}}{2(3)} = \frac{6 \pm \sqrt{36-12}}{6}$$

$$x = \frac{6 \pm \sqrt{24}}{6} = \frac{6 \pm \sqrt{4 \times 6}}{6} = \frac{6 \pm 2\sqrt{6}}{6}$$

$$x = \frac{2(3 \pm \sqrt{6})}{6} = \frac{3 \pm \sqrt{6}}{3}$$

$$S.S = \left\{ \frac{3 \pm \sqrt{6}}{3} \right\}$$

5. $x + \frac{1}{6x} = -\frac{5}{6}$

Soln: Multiplying both sides by $6x$ we get

$$x(6x) + 6x\left(\frac{1}{6x}\right) = 6x\left(-5/6\right)$$

$$6x^2 + 1 = -5x$$

$$\Rightarrow 6x^2 + 5x + 1 = 0$$

Here $a=6, \quad b=5, \quad c=1$

Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ —— (i)

Putting these values in Eq (i) we get

$$x = \frac{-5 \pm \sqrt{(5)^2-4(6)(1)}}{2\times 6} = \frac{-5 \pm \sqrt{25-24}}{12}$$

$$x = \frac{-5 \pm \sqrt{1}}{12} = \frac{-5 \pm 1}{12}$$

$$\Rightarrow x = \frac{-5+1}{12} \quad \text{or} \quad x = \frac{-5-1}{12}$$

$$x = \frac{-4}{12} = \frac{-1}{3} \quad \text{or} \quad x = \frac{-6}{12} = -\frac{1}{2}$$

$$S.S = \left\{-\frac{1}{3}, -\frac{1}{2}\right\}$$

6. $(x-1)(x+3) = 5(x+2)-3$

Soln: $(x-1)(x+3) = 5(x+2)-3$

$$x^2+3x-x-3 = 5x+10-3$$

$$x^2+2x-3 = 5x+7$$

$$x^2+2x-3-5x-7=0$$

$$x^2-3x-10=0$$

Here $a=1, \quad b=-3, \quad c=-10$

Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ —— (i)

Putting the values of a, b and c in Eq (i) we get;

$$x = \frac{-(-3) \pm \sqrt{(-3)^2-4(1)(-10)}}{2(1)} = \frac{3 \pm \sqrt{9+40}}{2}$$

$$x = \frac{3 \pm \sqrt{49}}{2} = \frac{3 \pm 7}{2}$$

So $x = \frac{3+7}{2}$ or $x = \frac{3-7}{2}$ $\qquad S.S = \{5, -2\}$.

$x = \frac{10}{2} = 5$ or $x = \frac{-4}{2} = -2$

(135)

7. $\frac{x-2}{x+2} = \frac{2x-3}{x-1}$

Soln:- By cross multiplication

$(x-2)(x-1) = (x+2)(2x-3)$

$\Rightarrow x^2 - x - 2x + 2 = 2x^2 - 3x + 4x - 6$

$x^2 - 3x + 2 = 2x^2 + x - 6$

$\Rightarrow 2x^2 + x - 6 - x^2 + 3x - 2 = 0$

$x^2 + 4x - 8 = 0$

Here; $a = 1$, $b = 4$, $c = -8$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ —— (i)

Putting the values of a, b and c in Eq (i)

$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-8)}}{2 \times 1} = \frac{-4 \pm \sqrt{16 + 32}}{2}$

$x = \frac{-4 \pm \sqrt{48}}{2} = \frac{-4 \pm \sqrt{16 \times 3}}{2} = \frac{-4 \pm 4\sqrt{3}}{2}$

$= \frac{-4}{2} \pm \frac{4}{2}\sqrt{3} = -2 \pm 2\sqrt{3}$

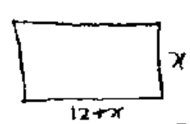
$S.S = \{-2 \pm 2\sqrt{3}\}$

Ex: 5.4

1. The area of a rectangular plot is 460 square meter. The length of plot is 12 m more than its breadth. Find the length and breadth of the plot.

Soln:- Let breadth of plot = x m

then length = $x + 12$ m



(136)

Area $= x(x+12) = x^2+12x$

But area is 460 m

$\Rightarrow x^2+12x = 460 \Rightarrow x^2+12x-460=0$

$x^2+32x-20x-460=0$

$x(x+32)-20(x+32)=0$

$\Rightarrow (x-20)(x+32)=0$

$\Rightarrow x-20=0$ or $x+32=0$

$x=20$ or $x=-32$

But $x=-32$ is not possible. So

$x=20$ m.

$\therefore$ Length $= 12+20 = 32$ m

Breath $= x = 20$ m

-460 ($32x$, -20)

2. The sum of positive number and its reciprocal is 2. Find the number.

Soln:: Let the number is $= x$

$\therefore$ Reciprocal $= \frac{1}{x}$

According to the condition: $x+\frac{1}{x}=2$

$\Rightarrow x.x+\frac{1}{x}.x = 2.x$ multiplying by x

$x^2+1=2x \Rightarrow x^2-2x+1=0$

$\Rightarrow (x)^2-2(x)(1)+(1)^2=0$

$\Rightarrow (x-1)^2=0$

$\Rightarrow \sqrt{(x-1)^2} = 0^2$

$\Rightarrow x-1=0$

$\Rightarrow x=1$

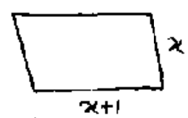
So the required number $=1$

(137)

3. The area of a board in a classroom is $12m^2$ if the length is 1m greater than its width, then find out the dimensions of the board.

Soln Let width of board = x m

$\therefore$ length $= x+1$ m



Area $= W \times L = x(x+1)\ m^2$

But given that area $= 12m^2$

So $x(x+1) = 12 \Rightarrow x^2+x = 12 \Rightarrow x^2+x-12=0$

$\Rightarrow x^2+4x-3x-12=0$ $\quad\therefore$ by factorizing

$x(x+4)-3(x+4)=0$

$(x-3)(x+4)=0$

$x-3=0$ or $x+4=0$

$x=3$ or $x=-4$

But $x=-4$ is not possible

So width $= x = 3$ m.

Length $= x+1 = 3+1 = 4$ m

4. Of Square of the lengths of an equilateral triangle is 4 times the length of its one side what will be the length of each side.

Soln: Let each side of triangle $= x$

Then according to the condition: $x^2 = 4x$

$\Rightarrow x^2-4x=0 \Rightarrow x(x-4)=0$

$\Rightarrow$ Either $x=0$ or $x-4=0 \Rightarrow x=4$

As $x=0$ is not possible $\Rightarrow x=4$

So length of each side $= 4$ m

5. The product of two consective integers is 20. Find it.

Soln Let one integer $= x$

Then 2nd " $= x+1$

According to the condition: $x(x+1) = 20$

$\Rightarrow x^2 + x = 20 \Rightarrow x^2 + x - 20 = 0$

$\Rightarrow x^2 + 5x - 4x - 20 = 0$

$x(x+5) - 4(x+5) = 0$

$(x-4)(x+5) = 0$

$\Rightarrow$ Either $x - 4 = 0$ or $x + 5 = 0$

$x = 4$ or $x = -5$

If $x = 4$, the numbers are: $4, 5$

If $x = -5$, the numbers are: $-5, -4$

6. Find two integers if their product is -8 and one integer is 6 less than the other.

Soln: Let first integer $= x$

2nd " $= x - 6$

According to the condition: Product $= -8$

$\Rightarrow x(x-6) = -8 \Rightarrow x^2 - 6x = -8$

$\Rightarrow x^2 - 6x + 8 = 0 \Rightarrow x^2 - 4x - 2x + 8 = 0$

$x(x-4) - 2(x-4) = 0$

$(x-2)(x-4) = 0$

$\Rightarrow x - 2 = 0$ or $x - 4 = 0$

$\Rightarrow x = 2$ or $x = 4$

If $x = 2$ then 2nd integer $= x - 6 = 2 - 6 = -4$

$\therefore$ $2, -4$ are the required integers.

If $x = 4$ then 2nd integer $= x - 6 = 4 - 6 = -2$

So $4, -2$ are the required integers

Ex : 5.5 (Objective Type Questions)

Q1 Encircle "T" if the statement is True and 'F' for false.

(i) The degree of a quadratic Equation is 2. (T)

(ii) In quadratic Equation $ax^2+bx+c=0$, $a=0$. (F)

(iii) The square of a number is always negative. (F)

(iv) If the product of two factors is zero. Then each factor equal to zero. (T)

(v) The general quadratic Eq contains three terms. (T)

2. Write a suitable answer in each blank space.

(i) A quadratic Eq is solved by <u>Three</u> Methods.

(ii) A quadratic Eq can be factorized into <u>Two</u> factors.

(iii) The formula used for solution of a quadratic Eq is called <u>Quadratic formula</u>

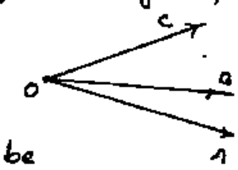
(iv) The values of variable which satisfy the quadratic equatic equation are called <u>Solution or root</u>.

3. Three options are given for each question. Choose the correct one.

(i) b (ii) (b) (iii) (a) (iv) b (v) (d)

Unit 7 FUNDAMENTALS OF GEOMETORY

Adjacent angles :- The two angles for which (i) Vertex is common (ii) One ray is common are called adjacent angles e.g in the given fig $\angle AOC$ and $\angle BOC$ are adjacent angles.

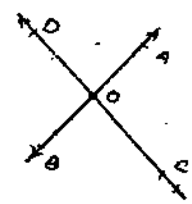


Supplementary angles

Two angles are said to be Supplementary angles if their sum is 180° e.g 120° and 60° are supplementary angles.

Complementary angles :- The two angles whose sum is 90° are called complementary angles e.g 40° and 50°

Vertically opposite angles :- If two lines intersect each other at a point then the pair of opposite angles are called vertically opposite angles. e.g in the given fig $\angle AOC$, $\angle BOD$ and $\angle BOC$, $\angle AOD$ are verticle angles. Noted that these verticle angles are congruent (equal)



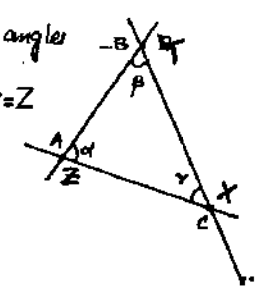
External - Internal angles of a triangle

Let ABC be a triangle then α, β, γ are its internal angles while X, Y, Z are called its external angles

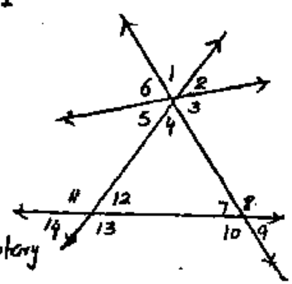
Note:- (i) $\alpha + \beta = X$, $\alpha + \gamma = Y$ and $\beta + \gamma = Z$

(ii) Sum of the internal angles of a triangle is always 180° i.e

$\alpha + \beta + \gamma = 180^\circ$



Ex: 7.1



1. From the adjoining figure:

(i) Write any two pairs of vertically opposite angles

(ii) Write any four pairs of adjacent angles.

(iii) Write any one pair of Complementary angles.

(iv) Write any four pairs of supplementary angles.

(v) Write any two pairs of adjacent angles that are neither Complementary nor supplementary

Soln:- (i) Two pairs of vertically opposite angles:

$(\angle 4, \angle 1)$ and $\angle 3, \angle 6$

(ii) $(\angle 1, \angle 2), (\angle 1, \angle 6), (\angle 11, \angle 12)$ and $\angle 11, \angle 14$ are the four pairs of adjacent angles.

(iii) $\angle 1, \angle 2$ is a pair of complementary angles.

(iv) $(\angle 7, \angle 8), (\angle 9 \angle 10), (\angle 11, \angle 12)$ are $(\angle 13, \angle 14)$ are pairs of Supplementary angles.

(v) $(\angle 2, \angle 3)$ and $(\angle 4, \angle 5)$ are pair of angles which are pair of angles neither complementary nor supplementary.

2. Find the supplement of the following angles:-

(i) 28°

Soln:- Suppose x is required Supplement Then

$x + 28^\circ = 180^\circ$

$x = 180^\circ - 28^\circ$

$\Rightarrow x = 152^\circ$

(ii) 86°

Soln:- Let the required Supp $= x$ then

$x + 86^\circ = 180^\circ$

$\Rightarrow x = 180^\circ - 86^\circ$

$x = 94^\circ$

(iii) 110°

Soln: Let the required supplement $= x$

then $x + 110^\circ = 180^\circ$

$\Rightarrow x = 180^\circ - 110^\circ = 70^\circ$

(iv) 154°

Soln: Let the required supplement $= x$

Then $x + 154^\circ = 180^\circ \Rightarrow x = 180^\circ - 154^\circ = 26^\circ$

3. Find the complement of following angles.

(i) 14°

Soln: Let the required comp $= x$

Then $x + 14^\circ = 90^\circ$

$\Rightarrow x = 90^\circ - 14^\circ$

$= 76^\circ$

(ii) 56°

Soln: Let the required complement $= x$

Then $x + 56^\circ = 90^\circ$

$\Rightarrow x = 90^\circ - 56^\circ$

$x = 34^\circ$

(iii) 33°

Soln: Let the required comp $= x$

$\Rightarrow x + 33^\circ = 90^\circ$

$\Rightarrow x = 90^\circ - 33^\circ = 57^\circ$

(iv) 78°

Soln: Let the required $= x$

Then; $x + 78^\circ = 90^\circ$

$\Rightarrow x = 90^\circ - 78^\circ$

$x = 12^\circ$

4. Find the measure of the following angles, if they are complement of each other.

(i) $5x+8,\ 7x+4$

Soln: As $5x+8$ and $7x+4$ are Supplement of each other

So $5x + 8 + 7x + 4 = 180$

$\Rightarrow (5x + 7x) + 8 + 4 = 180$

$\Rightarrow 12x + 12 = 180$

$\Rightarrow 12x = 180 - 12 = 168$

$\Rightarrow \frac{12x}{12} = \frac{168}{12}^{14} \Rightarrow x = 14$

So the required angles are:

$5x+8 = 5(14)+8 = 70+8 = 78^\circ$

$7x+4 = 7(14)+4 = 98+4 = 102^\circ$

(181)

(ii) $3x+9,\ 5x-5$

Soln:- As $3x+9$ and $5x-5$ are Supplement so,

$$3x+9+5x-5=180^\circ$$
$$\Rightarrow 3x+5x+9-5=180^\circ$$
$$\Rightarrow 8x+4=180 \Rightarrow 8x=180-4=176$$
$$\Rightarrow \frac{8x}{8}=\frac{176}{8}=22$$
$$\therefore 3x+9=3(22)+9=66+9=75^\circ$$
$$5x-5=5(22)-5=110-5=105^\circ$$

(iii) $4x+1,\ x+9$

Soln:- As $4x+1$ and $x+9$ are supplement so;

$$4x+1+x+9=180^\circ$$
$$\Rightarrow 4x+x+1+9=180^\circ \Rightarrow 5x+10=180^\circ$$
$$\Rightarrow 5x=180^\circ-10^\circ=170^\circ$$
$$\Rightarrow \frac{5x}{5}=\frac{170}{5} \Rightarrow x=34$$
$$\therefore 4x+1=4(34)+1=136+1=137^\circ$$
$$x+9=34+9=43^\circ$$

(iv) $x+3,\ 8x+15$

Soln:- As $x+3$ and $8x+15$ are supplement so;

$$x+3+8x+15=180^\circ$$
$$\Rightarrow (x+8x)+3+15=180^\circ$$
$$\Rightarrow 9x+18=180^\circ \Rightarrow 9x=180^\circ-18^\circ=162^\circ$$
$$\Rightarrow \frac{9x}{9}=\frac{162}{9} \Rightarrow x=18$$
$$\therefore x+3=18+3=21^\circ$$
and $8x+15=8(18)+15=144+15=159^\circ$

5. Find the angles given below if they are complement of each other.

(i) $2x+5,\ 3x-10$

Soln:- As $2x+5$ and $3x-10$ are complement so;

$$2x+5+3x-10=90^\circ$$

(182)

$\Rightarrow (2x+3x)+5-10=90^\circ$

$\Rightarrow 5x-5=90^\circ \Rightarrow 5x=90^\circ+5 \Rightarrow 5x=95$

$\Rightarrow \frac{5x}{5}=\frac{95}{5} \Rightarrow x=19$

$\therefore\ 2x+5=2(19)+5=38+5=43^\circ$

$3x-10=3(19)-10=57-10=47^\circ$

(ii) $4x+5,\ 2x-11$

Soln:- As sum of complementary angles $=90^\circ$

$\Rightarrow 4x+5+2x-11=90^\circ$

$(4x+2x)+5-11=90^\circ$

$\Rightarrow 6x-6=90^\circ \Rightarrow 6x=90+6=96$

$\Rightarrow \frac{6x}{6}=\frac{96}{6} \Rightarrow x=16$

$\therefore\ 4x+5=4(16)+5=64+5=69^\circ$

$2x-11 \Rightarrow 2(16)-11=32-11=21^\circ$

(iii) $x+9,\ 2x-15$

Soln As sum of two complementary angles $=90^\circ$

$\therefore\ x+9+2x-15=90^\circ$

$\Rightarrow x+2x+9-15=90^\circ$

$\Rightarrow 3x-6=90^\circ \Rightarrow 3x=90+6=96$

$\Rightarrow \frac{3x}{3}=\frac{96}{3} \Rightarrow x=32^\circ$

$\therefore\ x+9=32^\circ+9=41^\circ$

$2x-15=2(32)-15=64-15=49^\circ$

(iv) $8x+15,\ x+3$

Soln:- Sum of complentary angles $=90^\circ$

$\therefore\ 8x+15+x+3=90^\circ$

$\Rightarrow 8x+x+15+3=90$

$\Rightarrow 9x+18=90 \Rightarrow 9x=90-18=72$

$\Rightarrow \frac{9x}{9}=\frac{72}{9} \Rightarrow \boxed{x=8}$

$\therefore\ 8x+15=8(8)+15=64+15=79^\circ,\quad x+3=8+3=11^\circ$

(183)

6. Find the measure of the third angle of a triangle if the measure of each of its two angles are 37°, 56°

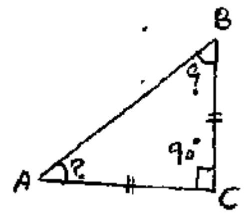
Sol:→ Let the measure of third angle $= x$

We know that sum of angles of triangle $= 180°$

$\therefore \quad x + 37° + 56° = 180°$

$\Rightarrow x + 93° = 180° \Rightarrow x = 180° - 93° = 87°$

7. A triangle is right angled isosceles triangle. Find the measure of its base angle.



Sol :- In given right angle triang ABC

$m\angle A + m\angle B + m\angle C = 180°$

$\Rightarrow m\angle A + m\angle B + 90° = 180°$

$\Rightarrow m\angle A + m\angle B = 180° - 90° = 90° \quad \text{—(i)}$

But $\quad m\overline{AC} = m\overline{BC}$

$\Rightarrow \quad m\angle A = m\angle B \qquad$ ∵ If two sides are equal the angle opposite to them are also.

Putting $m\angle A = m\angle B$ in (i)

$m\angle A + m\angle A = 90°$

$\Rightarrow 2m\angle A = 90° \Rightarrow \frac{2m\angle A}{2} = \frac{90°}{2}$

$\Rightarrow m\angle A = 45°$

So $\quad m\angle A = m\angle B = 45°$

8. Angles of a triangles in 2:3:4. Find their measure.

Sol:. As sum of three angle of triangle $= 180°$

ratio b/w sides $= 2:3:4$

$\therefore 2x + 3x + 4x = 180 \Rightarrow 9x = 180°$

$\Rightarrow \frac{9x}{9} = \frac{180}{9} \Rightarrow x = 20$

$\therefore$ One angle $= 2x = 2(20) = 40°$

2nd angle $= 3x = 3(20) = 60°$

3rd angle $= 4x = 4(20) = 80°$

9. In right angled triangle, the accute angles are in in 5:13. Find the measures of these angles.

Soln:- The sum of accute angles of right triangle = $90°$

Ratio b/w the angles = $5:13$

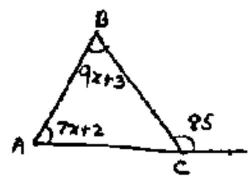
Sum of ratio $= 5+13 = 18$

$\therefore$ First angle $= \frac{5}{18} \times 90 = 25°$

2nd angle $= \frac{13}{18} \times 90 = 65°$

10. Measures of exterior angle of a triangle is 85. Measures of its non adjacent interior angles are $7x+2$ and $9x+3$ degrees. Find the measures of interior angles.

Soln:- As the sum of non adjacent interior angle = Exterior angle



$\therefore$ $7x+2+9x+3 = 85$

$\Rightarrow 16x + 5 = 85$

$\Rightarrow 16x = 85 - 5 = 80$

$\Rightarrow \frac{16x}{16} = \frac{80}{16} \Rightarrow x = 5°$

$\therefore$ one interior angle $= 7x+2 = 7(5)+2 = 35+2 = 37°$

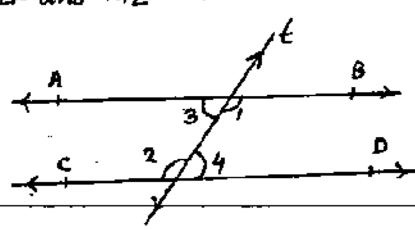
2nd angle $= 9x+3 = 9(5)+3 = 45+3 = 48°$

3rd angle $= 180 - (37+48) = 180 - 85 = 95°$

Alternate angles :- Suppose $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two parallel lines and 't' is a transversal. Then $\angle 1, \angle 2$ and $\angle 3, \angle 4$ are alternate angles.

They are equal in measures i.e

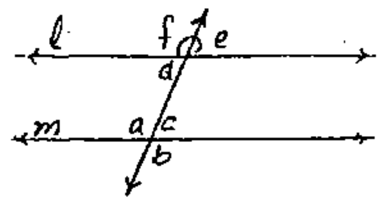
$m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$



(185)

EX: 7.2

1. In the given figure, $l \parallel m$ and $a = 110^\circ$. Find the unkno...



Soln:- As 'a' and 'b' are verticle opposite angles $\therefore a = b$

$\Rightarrow \boxed{b = a = 110^\circ}$

$\because m\angle a + m\angle c = 180^\circ$ because a and c are Supplem...

$\Rightarrow 110 + c = 180^\circ$

$\Rightarrow c = 180^\circ - 110^\circ = 70^\circ$

so $\boxed{c = 70^\circ}$

As c and d are alternate angles so They must equal

$\Rightarrow \boxed{c = d = 70^\circ}$ i.e $c = 70^\circ$

$\because$ d and e are verticle opposite so

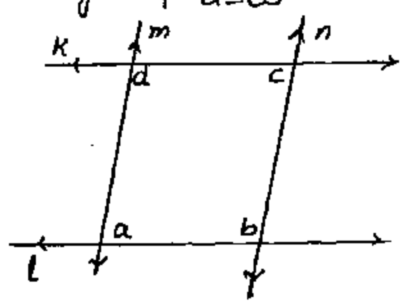
$e = d = 70^\circ \Rightarrow \boxed{e = 70^\circ}$

$\because$ e and f are supplementary so

$e + f = 180 \Rightarrow f + 70 = 180$

$\Rightarrow f = 180 - 70 = 110 \Rightarrow \boxed{f = 110^\circ}$

2. In the given figure $k \parallel l$ and $m \parallel n$ find the measures of unknown angles. If $a = 65^\circ$



Soln:- As sum of interior angles one side on traversal = 180

$\Rightarrow a + d = 180^\circ$

$\Rightarrow 65^\circ + d = 180^\circ$

$\Rightarrow d = 180^\circ - 65^\circ = 115^\circ$

(186)

Similarly $a + b = 180°$

$\Rightarrow 65° + b = 180° \Rightarrow b = 180° - 65° = 115°$

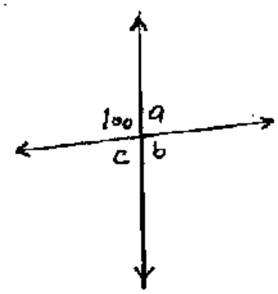
And $d + c = 180°$

$115° + c = 180°$

$\Rightarrow c = 180° - 115° = 65°$

3. Find the unknown angles in the following figures:

(i) Solns.



$b = 100°$ ∵ b is verticle to 100

$c + 100° = 180°$ ∵ Supplementary angles

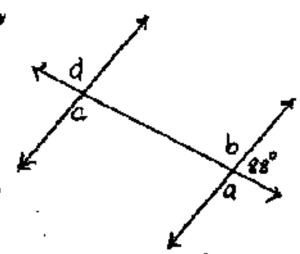
$\Rightarrow c = 180° - 100°$

$c = 80°$

As a and c are opposite verticle

$\therefore a = c = 80° \Rightarrow a = 80°$

(ii)



As b and 88° are supplement of each other

$\therefore b + 88° = 180°$

$\Rightarrow b = 180° - 88°$

$b = 92°$

Now b and a verticle opposite angle

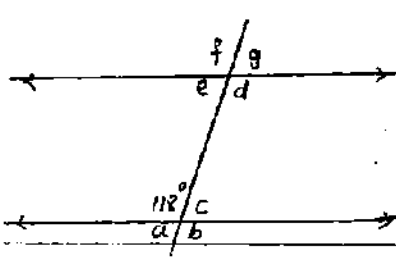
$\therefore a = b = 92° \Rightarrow a = 92°$

As b and c are alternate angles which are always equal

$\therefore c = b = 92° \Rightarrow c = 92°$

Now c and d are verticle $\Rightarrow d = 92°$

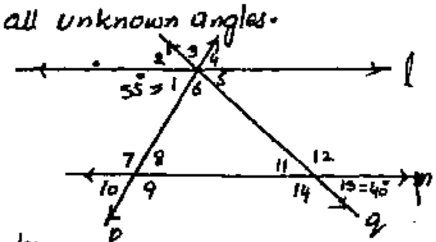
(iii)



Sol:-

	Reasons:
$b = 118°$	They are verticle opposite angle
$a + 118° = 180°$	Supplementary angles
$\Rightarrow a = 180° - 118° = 62°$	
$a = c = 62°$	verticle opposite angle.
$e = c \Rightarrow e = 62°$	e and c are alternate angles
$d = 118°$	∵ Alternate angles.
$f = d = 118°$	∵ f and d are verticle angle
$g = e = 62°$	∵ g an e are verticle angle.

4. In the given figure $l \parallel m$, $m\angle 1 = 55°$ and $m\angle 13 = 40°$. Find the measures of all unknown angles.



Sol:-
As ∠1 and ∠4 are verticle opposite angle

∴ $\boxed{m\angle 4 = m\angle 1 = 55°}$

As ∠1 and ∠8 are verticle angles.

∴ $\boxed{m\angle 8 = m\angle 1 = 55°}$

∠8 and ∠10 are verticle opposite angles

∴ $\boxed{m\angle 10 = m\angle 8 = 55°}$

As $m\angle 8 + m\angle 9 = 180°$ ∵ ∠9, ∠8 are supplementary angles

$\Rightarrow 55° + m\angle 9 = 180°$

$\Rightarrow m\angle 9 = 180° - 55° = 125°$

$\Rightarrow \boxed{m\angle 9 = 125°}$

Similarly ∠9 and ∠7 are verticle opposite angles

$\Rightarrow m\angle 7 = m\angle 9 = 125° \Rightarrow \boxed{m\angle 7 = 125°}$

(188)

As $\angle 5$ and $\angle 13$ are verticle angles $\Rightarrow$ $\boxed{m\angle 13 = m\angle 5 = 40^\circ}$

$\angle 13$ and $\angle 11$ are verticle opposite angles. Therefore;

$\boxed{m\angle 11 = m\angle 13 = 40^\circ}$

$\because m\angle 12 + m\angle 13 = 180^\circ$. As they are supplementary angle

$\Rightarrow 40^\circ + m\angle 12 = 180^\circ$

$\Rightarrow m\angle 12 = 180^\circ - 40^\circ$

$\boxed{m\angle 12 = 140^\circ}$

$\Rightarrow \boxed{m\angle 14 = 140^\circ}$ $\because$ $\angle 14$ and $\angle 12$ are opposite verticle

$\boxed{m\angle 2 = 40^\circ}$ $\because$ $\angle 2$ and $\angle 5$ are verticle opposite

As $m\angle 2 + m\angle 3 + m\angle 4 = 180^\circ$

$\Rightarrow 40^\circ + m\angle 3 + 55^\circ = 180^\circ \Rightarrow m\angle 3 = 180^\circ - 40^\circ - 55^\circ$

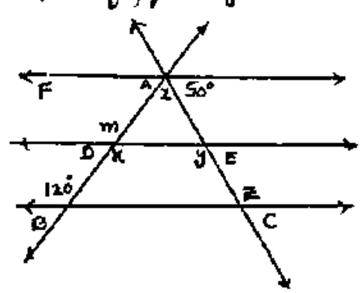
$\boxed{m\angle 3 = 85^\circ}$

As $\angle 3$ and $\angle 6$ are verticle opposite so;

$m\angle 6 = m\angle 3 = 85^\circ \Rightarrow \boxed{m\angle 6 = 85^\circ}$

5. In the figure $\overleftrightarrow{BC} \parallel \overleftrightarrow{DE} \parallel \overleftrightarrow{FA}$. Find the measures of Unknown angles k, m, x, y and z. If $\overleftrightarrow{AB}$ and $\overleftrightarrow{BD}$ are equal then find another pair of equal segments.

Soln:-



As $\overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$

$\Rightarrow \boxed{m\angle k = 120^\circ}$

As $\angle k$ and $\angle m$ are verticle opposite angles

$\therefore m\angle m = m\angle k = 120^\circ$

$\Rightarrow \boxed{m\angle m = 120^\circ}$

As $\overleftrightarrow{DE} \parallel \overleftrightarrow{FA} \Rightarrow m\angle m = \angle x + 50^\circ$

$\Rightarrow m\angle x = m\angle m - 50^\circ = 120^\circ - 50^\circ = 70^\circ$

(189)

$\boxed{m\angle E = 50°}$ ∵ $\angle E$ and $50°$ are interior angle of tranversal.

As $m\angle E + m\angle y = 180°$

$\Rightarrow 50 + m\angle y = 180° \Rightarrow m\angle y = 180° - 50°$

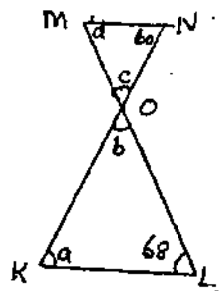
$\Rightarrow \boxed{m\angle y = 130°}$

As $\angle y$ and $\angle z$ are alternate angles so;

$\Rightarrow m\angle z = m\angle y = 130° \Rightarrow \boxed{m\angle z = 130°}$

6. $\overline{AE}$ and $\overline{EC}$ are equal segments.

In the given figure $\overline{KL} \parallel \overline{MN}$. Find the measure of angles a, b, c and 'd'.



Soln: As $\overline{KL} \parallel \overline{MN}$

$\Rightarrow$ Alternate angle must equal

∴ $m\angle a = 60°$ and $m\angle d = 68°$

Now In $\triangle MNO$,

$m\angle c + m\angle d + 60 = 180°$

$\Rightarrow m\angle c + 68 + 60 = 180$

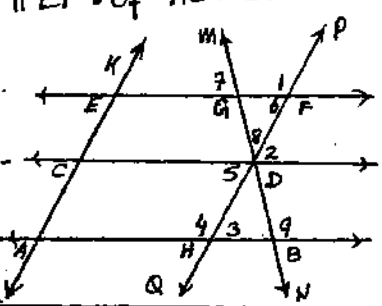
$\Rightarrow m\angle c = 180 - 68 - 60$

$m\angle c = 52°$

∵ But $m\angle c = m\angle b \Rightarrow \boxed{m\angle b = 52°}$

In the figure $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$. If $\overline{AC} = \overline{CE}$ Name two other pairs of congruent line segments.

Also find the measures of unknown angles when $m\angle 1 = 100°$ and $m\angle 7 = 70°$



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Two pair of congruent segments are : $m\overline{HD} = m\overline{DF}$
$m\overline{BD} = m\overline{DG}$

$\because m\angle 1 + m\angle 6 = 180^\circ$

$\Rightarrow 100 + m\angle 6 = 180^\circ$ $\quad \because m\angle 1 = 100^\circ$

$\Rightarrow m\angle 6 = 180^\circ - 100^\circ = 80^\circ$

But $m\angle 2 = m\angle 6 = 80^\circ$ $\quad \because \angle 2, \angle 6$ are alternate angles.

$\Rightarrow \boxed{m\angle 2 = 80^\circ}$

$\because m\angle 5 = m\angle 2 = 80^\circ$ $\quad \because \angle 2, \angle 5$ are verticle opposite

$\Rightarrow \boxed{m\angle 5 = 80^\circ}$

As $\angle 3$ and $\angle 5$ are alternate angles. Therefore;

$m\angle 3 = m\angle 5 = 80^\circ \Rightarrow \boxed{m\angle 3 = 80^\circ}$

As $m\angle 3 + m\angle 4 = 180^\circ$

$\Rightarrow 80^\circ + m\angle 4 = 180^\circ$

$\Rightarrow m\angle 4 = 180^\circ - 80^\circ = 100^\circ \Rightarrow \boxed{m\angle 4 = 100^\circ}$

As $m\angle 7 + m\angle G = 180^\circ$

$\Rightarrow m\angle G = 180^\circ - m\angle 7 = 180^\circ - 70^\circ = 110^\circ$

$\Rightarrow \boxed{m\angle G = 110^\circ}$

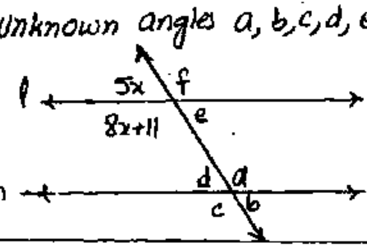
But $m\angle G = m\angle 9$ $\quad \because$ Alternate angles

$\Rightarrow \boxed{m\angle 9 = 110^\circ}$

Also $m\angle 2 + m\angle 8 = m\angle 9$

$\Rightarrow m\angle 8 = m\angle 9 - m\angle 2 = 110^\circ - 80^\circ = 30^\circ$

8. Find the measures of unknown angles a, b, c, d, e and f when $l \parallel m$



Soln

As $5x$ and $8x+11$ are supplementary angles

$\Rightarrow 5x + 8x + 11 = 180$

$\Rightarrow 13x = 180 - 11 = 169$

$\Rightarrow \frac{13x}{13} = \frac{169}{13} \Rightarrow \boxed{x = 13}$

Now $m\angle f = 5x + 11$ ∵ opposite verticle angle

$m\angle f = 5(13) + 11 = 65 + 11 = 76°$

and $m\angle e = 5x = 5(13) = 65°$

As $\angle e$ and $\angle d$ are alternate angles

$\Rightarrow m\angle d = m\angle e = 65°$

$\Rightarrow m\angle b = 65°$ ∵ $\angle d$, $\angle b$ are verticle opposite angle

Now $m\angle d + m\angle c = 180°$

$65 + m\angle c = 180°$

$\Rightarrow m\angle c = 180° - 65° = 115°$

As $\angle a$ and $\angle c$ are verticle opposite angles, so

$m\angle a = m\angle c = 115°$

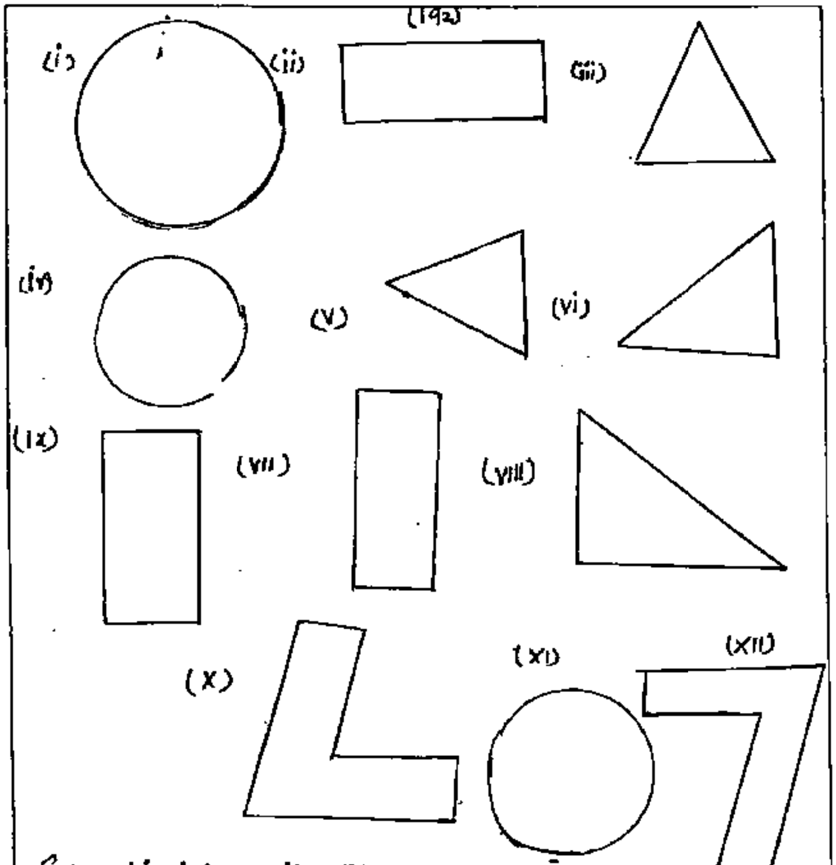
EX: 7.3

1. Which of the followings are congruent and which are similar?

Note:- (i) The figures having same shape and same size are called congruent figures

(ii) The geometrical figure having same shape but having different size are called similar figure.

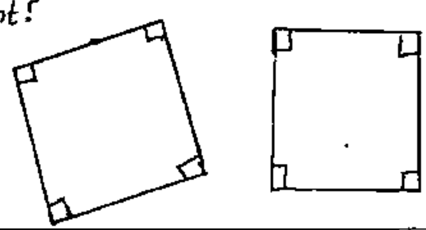
(192)



Soln (i, iv), (ii, vii) and (x, xii) are Congruent while (vi, viii), (ii, vii, ix) and (i, iv, xi) are similer.

2. Which of the following figure are similer and which are congruent?

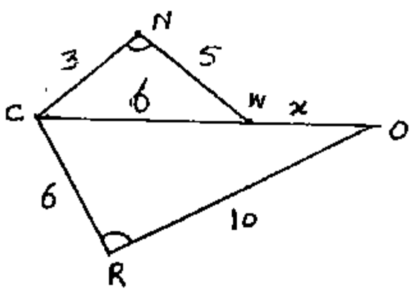
Soln:- (i) They are Congruent.



(ii) They are Similer.
(iii) They are Similer.
(iv) They are

3. Are the triangles given in each figure Similer. If yes, find the unknown elements.

(i)



Soln. Yes, they are similer by S.A.S

Now, as They are similer triangle. therefore;

$\frac{3}{6} = \frac{5}{10} = \frac{6}{6+x}$

$\Rightarrow \frac{1}{2} = \frac{1}{2} = \frac{6}{6+x} \Rightarrow \frac{6}{6+x} = \frac{1}{2}$

$\Rightarrow 6+x = 2\times 6$ by cross multiplication

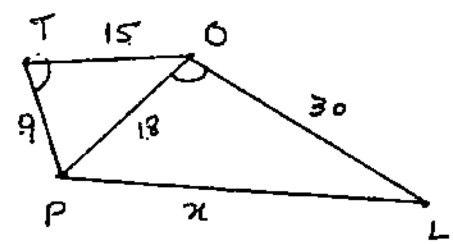
$6+x = 12$

$\Rightarrow x = 12-6 \Rightarrow \boxed{x=6}$

(ii)

Soln

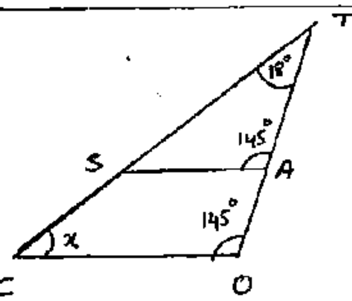
Yes, ΔPOL and ΔPOT are Similer by S.A.S



$\therefore \frac{9}{18} = \frac{15}{30} = \frac{18}{x}$

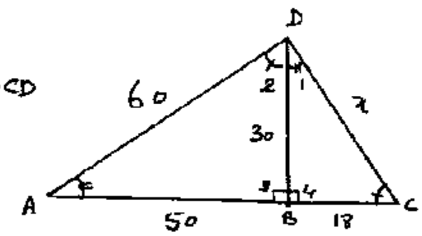
$\Rightarrow \frac{1}{2} = \frac{1}{2} = \frac{18}{x} \Rightarrow \frac{18}{x} = \frac{1}{2} \Rightarrow x = 36$

(iii)
Soln:-
Yes ΔCOT and ΔSAT are similer by A.A.A
Now in ΔCOT,



$x + 145° + 18° = 180°$
$\Rightarrow x + 163 = 180°$
$\Rightarrow x = 180° - 163° = 17°$

(iv)
Soln:- In $\Delta ABD \leftrightarrow \Delta BCD$
$m\angle A = m\angle 1$
$m\angle 2 = m\angle C$
$m\angle 3 = m\angle 4$



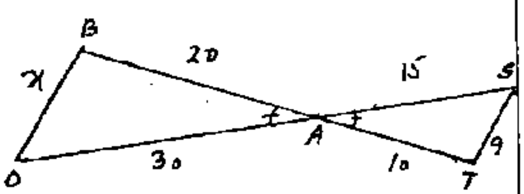
∴ Both triangles are Similer by A.A.A
Therefore the ratio of lengths of the corresponding sides must equal

$\Rightarrow \frac{x}{60} = \frac{18}{30} = \frac{30}{50} \Rightarrow \frac{x}{60} = \frac{3}{5} = \frac{3}{5}$
$\Rightarrow \frac{x}{60} = \frac{3}{5} \Rightarrow \frac{x}{60} \cdot 60 = \frac{3}{5} \times 60$

$\boxed{x = 36}$

So $x = 36$

(V)
Soln:- In $\Delta AOB \leftrightarrow \Delta AST$
$\overline{OA}$ and $\overline{AS}$ are proportional
$\overline{BA}$ and $\overline{AT}$ are proportional. Also there included angle are congruent.



$\Rightarrow$ ΔAOB and ΔAST are similer. by S.A.S

As they are similer so the ratio b/w there sides will same i.e

$$\frac{15}{30} = \frac{10}{20} = \frac{9}{x}$$

$$\Rightarrow \frac{1}{2} = \frac{1}{2} = \frac{9}{x} \Rightarrow \frac{9}{x} = \frac{1}{2} \Rightarrow \boxed{x=18} \quad \because \text{Cross multiply}$$

<u>Congruent triangless</u>- Two triangles are said to be Congruent if they have same shape and same size. There are six components of a triangle. Three are angles and three of them are sides. If all the six components of both the triangle have same measure then they will be congruent.

Now the possible cases of congruent triangles are:

(i) If all the sides of one triangle are equal to the corresponding sides of the other triangle then both will congruent and is known as S.S.S ≅ S.S.S postulate

(ii) If two sides and included angle of a triangle are equal in size to the corresponding angle and sides of the other triangle, they will be congruent and known as S.A.S ≅ S.A.S postulate.

(iii) If two angles and included side of one triangle are equal to the corresponding sides and angle, both the triangle will be congruen. (A.S.A ≅ A.S.A)

(iv) If in a right angled triangle one side and a hypotenous are equal in size to the side and hypotenous of the other triangle, they will again be congruent (H.S ≅ H.S)

(196)

(Ex# 7.4)

1. Consider the marked element of the triangles and show how are the triangle congruent

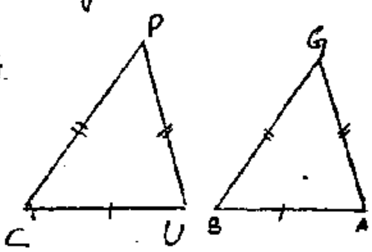
(i) Soln:- In $\triangle PCU \leftrightarrow \triangle ABG$

$m\ \overline{PC} = m\ \overline{BG}$

$m\ \overline{CU} = m\ \overline{AB}$

$m\ \overline{PU} = m\ \overline{AG}$

$\Rightarrow \triangle PCU$ and $\triangle ABG$ are congruent by S.S.S postulate.



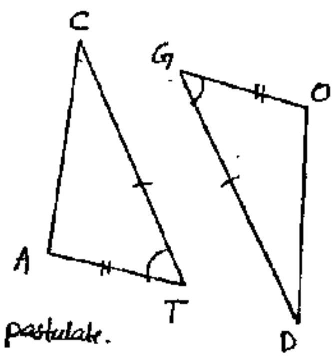
(ii) In $\triangle ACT \leftrightarrow \triangle DOG$

$m\overline{AT} = m\overline{DG}$

$m\angle T = m\angle G$

$m\ \overline{CT} = m\ \overline{DG}$

$\Rightarrow \triangle ACT$ and $\triangle DOG$ are congruent by S.A.S postulate.



(iii) Soln:-

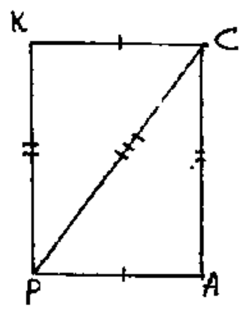
In $\triangle APC \leftrightarrow \triangle PKC$

$m\overline{AP} = m\ \overline{CK}$

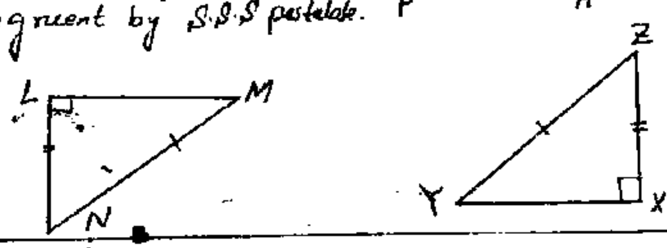
$m\ \overline{AC} = m\ \overline{PK}$

$m\ \overline{PC} = m\overline{PC}$

$\Rightarrow$ both the triangles are congruent by S.S.S postulate.



(iv)



197

Soln:- In $\Delta LMN \longleftrightarrow \Delta XYZ$

$m\overline{LM} = m\overline{XZ}$

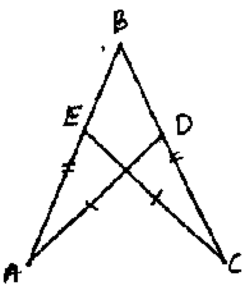
$m\overline{MN} = m\overline{YZ}$

both are right angled triangle.

So ΔLMN and ΔXYZ are congruent by H.S.

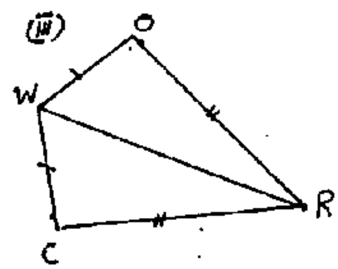
2. In the following figures, some of the elements of the triangle have been marked to show that they are congruent. Write at least one corresponding element which when combines with the given information makes the triangle congruent.

(i)



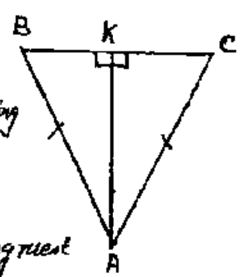
The corresponding $m\angle B \cong m\angle B$ makes ΔADB Congruent to ΔABE

(iii)



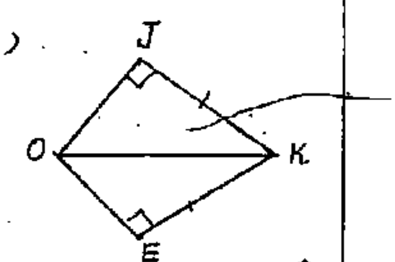
The Corresponding $m\overline{WR} = m\overline{WR}$ makes ΔCRW and ΔORW congruent i.e $\Delta CRW \cong \Delta ORW$.

(ii)



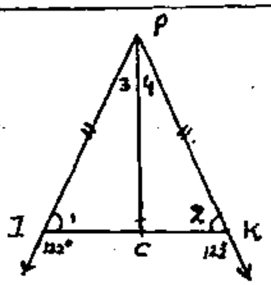
The Corresponding $m\overline{AK} = m\overline{AK}$ makes The triangle congruent i.e $\Delta ABK \cong \Delta ACK$ by H.S

(iv)



The corresponding $m\overline{OK} \cong m\overline{OK}$ makes $\Delta KOJ \cong \Delta KOE$ by $H.S \cong H.S$

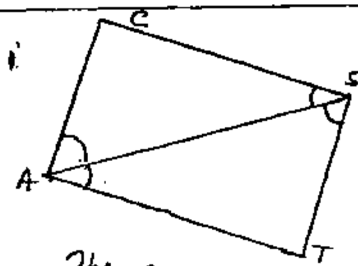
(v)



The corresponding $m\overline{PC} = m\overline{PC}$ makes $\Delta PCI \cong \Delta PKC$

OR The corresponding $m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$ makes $\Delta PCI \cong \Delta PKC$

i



The corresponding $m\overline{AS} \cong m\overline{AS}$ makes the $\Delta ACS \cong \Delta STA$ by A.S.A

Quadrilaterals :- A four sided closed figure is called quadrilateral. It has many types.

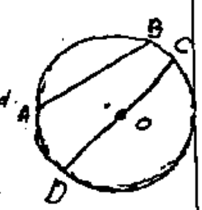
(i) If all the sides of a quadrilateral are equal and each angle is right angle, it is called a Square.

(ii) If the opposite sides of a quadrilateral are equal and each angle is right angle, it is called rectangle.

(iii) If opposite sides and opposite angle are congruent, the quadrilateral is called parallelogram.

Circle:- The set of points on plane which are equidistant from a fixed point is called circle. The fixed point is called its circle.

Chord :- A line which cuts the circle two points e.g in the fig $\overleftrightarrow{AB}$ is chord.



Diameter:- A chord passing through the centre of circle is called diameter e.g $\overline{CD}$ is diameter

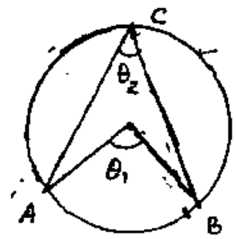
(199)

Arc of a circle:- A portion of a circle joining any two points A and B of a circle is called an arc and denoted by $\widehat{AB}$.

The arc in a semi-circle is called minor arc. while the arc in a whole circle is called major arc. In the given circle $\widehat{AOB}$ is minor arc and $\widehat{ACB}$ is major arc.

Noted that the central angle of minor arc is double in measure to that of major arc.

i.e $\theta_1 = 2\theta_2$.



Tangent to a circle:- A line which touch the circle at single point is called tangent to a circle.

EXCERCISE: 7.5

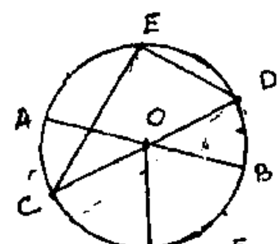
1. Name the centre of the given circle. Name 2 chords, and 2 diameters. Also name a radial segment shown in circle.

Soln:- (i) Centre of circle = O

(ii) Diameters of circle = $\overline{AB}$ and $\overline{CD}$

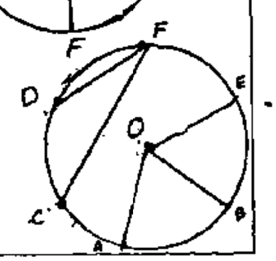
(iii) Chords of circle = $\overline{CE}$, $\overline{ED}$

(iv) Radial Segment = $\overline{OF}$



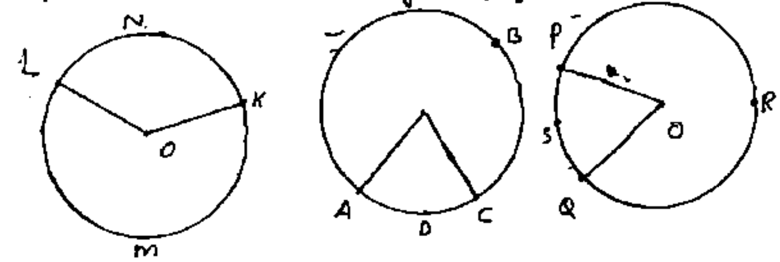
2. Name the radii and chords in this figure. Also name the concyclic points.

Soln:- (i) Radii of circle are: $\overline{OA}$, $\overline{OB}$ and $\overline{OE}$



(ii) Concyclic points are: A, B, C, D, E, F

3. Name the major and minor arc in the given circles. Also name the sectors in given figure.



Soln: (i) Minor arc = $\widehat{KNL}$, Major Arc = $\widehat{KML}$

Sector = Region bounded by radial segments and minor arc i.e region bounded by $\overline{OK}$, $\overline{OL}$ and $\widehat{KL}$

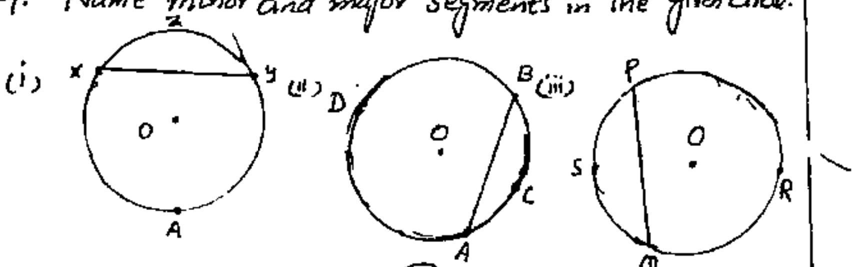
(ii) minor arc = $\widehat{ADC}$ and Major arc = $\widehat{ABC}$

Sector = Region bounded by $\overline{OA}$, $\overline{OC}$ and $\widehat{AC}$

(iii) Minor arc = $\widehat{PSQ}$, Major arc = $\widehat{PRQ}$

Sector = bounded by $\overline{OQ}$, $\overline{OP}$ and $\widehat{PQ}$

4. Name minor and major Segments in the given circle.



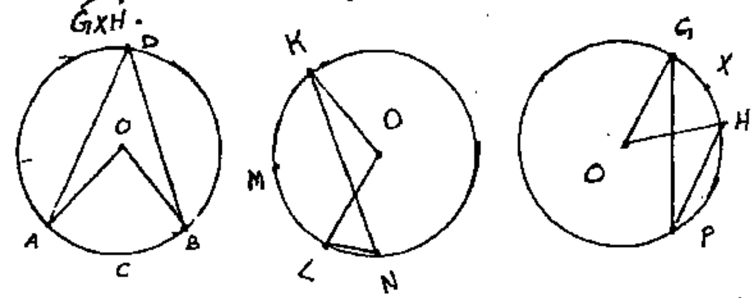
Soln: (i) Minor Segment: $\widehat{XZY}$, Major Segment $\widehat{XYA}$

(ii) Minor Segment $\widehat{ABC}$, major Segment $\widehat{ABD}$

(iii) Minor Segment $\widehat{PQS}$, major Segment $\widehat{PQR}$

(201)

5. Name the central angles of arcs $\widehat{ABC}$, $\widehat{KLM}$ and $\widehat{GXH}$.



Also name the inscribed angles in the corresponding major arcs.

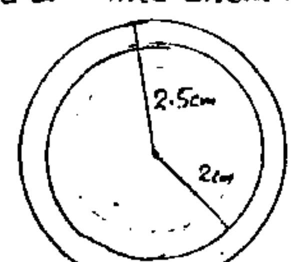
Soln (i) The central angle of arc $\widehat{ACB}$ = $\angle AOB$.
The inscribed angle of major arc = $\angle ADB$

(ii) Central angle of $\widehat{KML}$ = $\angle KOL$
The inscribed angle of major arc = $\angle KNL$

(iii) Central angle of $\widehat{GXH}$ = $\angle GOH$
Inscribed angle major arc = $\angle GPH$

6. Draw the concentric circles of radii 2cm, 2.5cm and 3cm. What is the diameter of each circle

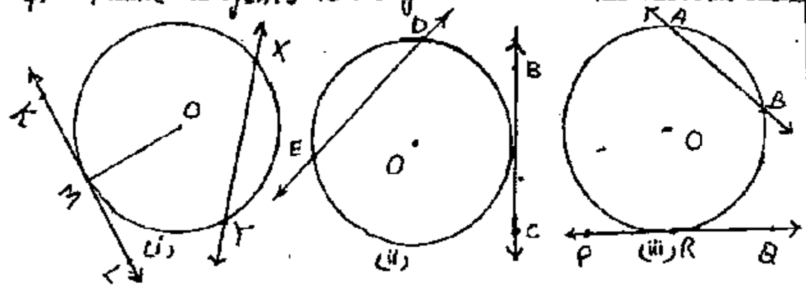
Soln: The concentric circles are:



Note:- Those circles whose centre is same are called concentric circles.

As diameter = $2r$ ⇒ diameter of circle of $r = 2cm$ is $2(2) = 4cm$
diameter of circle of $r = 2.5cm$ is $= 2(2.5) = 5$
diameter of circle of 3cm radii $= 3(2) = 6cm$

7. Name tangents to the given circles. Also name the secants.

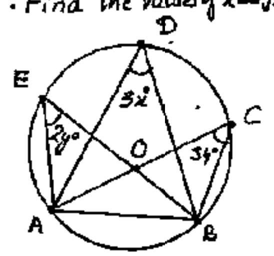


Soln:- (i) Tangent = $\overline{KL}$, Secant = $\overline{XY}$

(ii) Tangent = $\overline{BC}$, Secant = $\overline{DE}$

(iii) Tangent = $\overline{PQ}$, Secant = $\overline{AB}$

8. In the given figure, $m\angle ACB = 54°$. Find the values of x and y.



Soln:- In the given figure,

$m\angle AEB = m\angle ACB$

$\Rightarrow 2y = 54°$

$\Rightarrow y = \frac{54°}{2} = 27°$

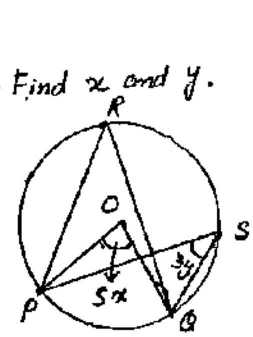
So $\boxed{y = 27°}$

As $m\angle ADB = m\angle ACB$

$\Rightarrow 3x = 54° \Rightarrow x = \frac{54}{3} = 18°$

So $\boxed{x = 18°}$

9) In the given figure $m\angle PRQ = 45°$. Find x and y.



Soln:- As $\angle PRQ$ is the central angle of major arc and $\angle POQ$ is the central angle of minor arc.

But we know that $m\angle POQ = 2m\angle PRQ$

$\Rightarrow \quad 5x = 2(45) = 90$

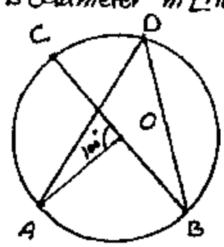
$\Rightarrow \quad \frac{5x}{5} = \frac{90}{5} \Rightarrow \boxed{x = 18}$

As $\angle PSQ$ and $\angle PRQ$ are the central angles of minor arcs Therefore, $m\angle PSQ = m\angle PRQ$.

$\Rightarrow \quad 3y = 45 \Rightarrow y = \frac{45}{3} = 15$

$\therefore \boxed{y = 15^\circ}$

10. "O" is the centre of circle. $\overline{BC}$ is diameter $m\angle AOC = 100^\circ$. Find $m\angle ADB$



Soln:- As,

$m\angle AOC + m\angle AOB = 180^\circ$

$\Rightarrow \quad 100^\circ + m\angle AOB = 180^\circ$

$\Rightarrow \quad m\angle AOB = 180^\circ - 100^\circ$

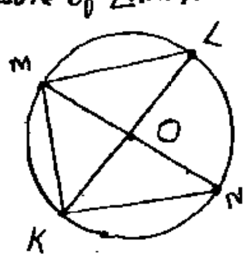
or $\quad m\angle AOB = 80^\circ$

But $\angle AOB$ = Central angle of minor arc, $\angle ADB$ = Central angle of major arc $\Rightarrow m\angle AOB = 2m\angle ADB$

$\Rightarrow \quad 80^\circ = 2m\angle ADB$

$\Rightarrow \quad m\angle ADB = \frac{80}{2} = 40^\circ$

11. In the figure, O is the centre of circle. $\overline{MN}$ is the bisector of $\angle KML$. Find the measure of $\angle KNM$.



Soln:- $\because$ Angle in semi-circle is always 90°

$\therefore \quad m\angle KML = 90^\circ$

Also $\overline{MN}$ is the bisector of $\angle KML$

$\Rightarrow \quad m\angle KNM = \frac{1}{2} m\angle KML = \frac{90}{2} = 45^\circ$

$\Rightarrow \quad m\angle KNM = 45^\circ$

Excercise 7 (objective type)

Four suggested answers are given select the Correct.

Ans

(1) d	2. c	3. b
4. a	5. c	6. b
7. b	8. d	9. d
10. c	11. a	12. a
13. a	14. b	15. d
16. d	17. c	18. b
19. d	20. a	21. a
22. c	23. b	24. d
25. b	26. c	27. b
28. b	29. d	30 d
31. b	32. c	33. a

(205)

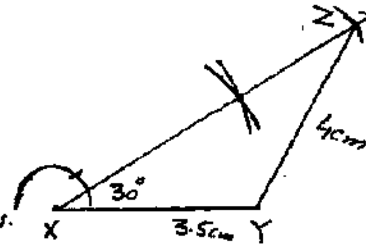
UNIT 8: PRACTICAL GEOMERY

EXERCISE 8.1

1. Construct ΔXYZ, for the following assumptions:

(i) $m\angle X = 30^\circ$, $m\overline{XY} = 3.5\,cm$, $m\overline{XZ} = 4cm$

Sol: Steps of constructions:



(i) Draw a line segment $\overline{XY}$ of length 3.5cm.

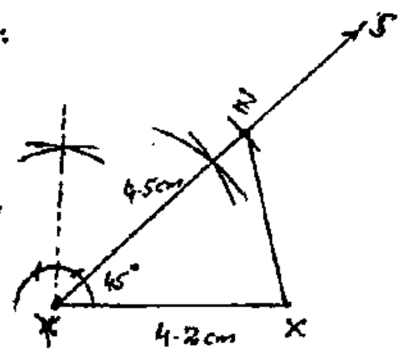
(ii) With centre X, draw angle of measure 30° with compass.

(iii) centre Y, draw an arc of length 4cm which cuts $\overrightarrow{XR}$ at point Z. Join Y to Z

(iv) XYZ is the required triangle.

II. $m\angle Y = 45^\circ$, $m\overline{XY} = 4.2cm$, $m\overline{YZ} = 4.5cm$

Sol: Steps of Constructions:



(i) Draw a line segment $\overline{XY}$ of length 4.2 cm.

(ii) With centre Y Draw angle of 45°

(iii) With centre Y draw an arc of length 4.5cm which cuts $\overrightarrow{YS}$ at Z.

(iv) Join X to Z.
XYZ is the required triangle.

(iii) $m\angle Z = 60^\circ$, $m\overline{XZ} = 3.8cm$, $m\overline{YZ} = 4.4cm$

Sol: Steps of constructions:

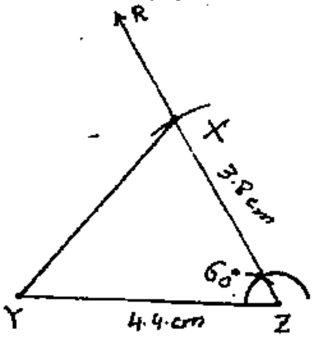
(i) Draw a line segment of $m\overline{YZ} = 4.4$ cm

(ii) With centre Z, Construct angle of measure 60°

(iii) With centre Z, draw an arc of length 3.8cm which cuts $\overrightarrow{ZR}$ at X.

(iv) Join Y to X.

XYZ is the required triangle.



2. Construct a ΔABC, for the following assumptions:

(i) $m\overline{AB} = 4.5cm$, $m\angle A = 45^\circ$ and $m\angle B = 60^\circ$

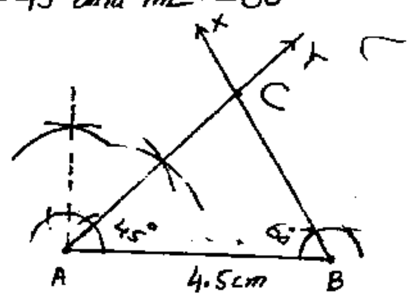
Sol: Steps of Constructions.

(i) Draw a line segment $m\overline{AB} = 4.5cm$

(ii) Construct angle 45° with centre A.

(iii) With centre B, Construct angle of measure 60° which cuts $\overrightarrow{AY}$ at C.

ABC is the required triangle



(ii) $m\overline{BC} = 5cm$, $m\angle B = 30^\circ$ and $m\angle C = 75^\circ$

Soln: STEPS OF Constructions

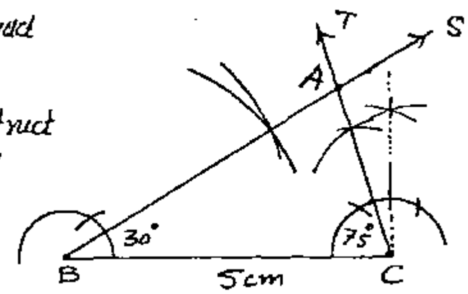
(i) Draw $m\overline{BC} = 5cm$.

(207)

(ii) With centre B, construct angle of measure 30°

(iii) With centre C, Construct angle of measure 75° which cuts $\overrightarrow{BS}$ at A.

Hence ABC is the required triangle.

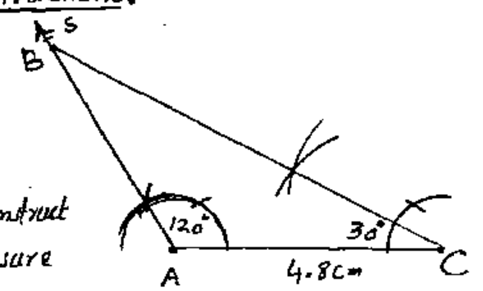


(iii) $m\overline{AC} = 4.8cm$, $m\angle A = 120^\circ$ and $m\angle C = 30^\circ$

Sol: <u>Steps of Constructions:</u>

(i) Draw a line-segment $\overline{AC}$ of measure 4.8 cm.

(ii) With centre A, Construct an angle of measure 120°



(iii) With centre C, construct an angle of measure 30° which meets $\overrightarrow{AS}$ at B. Hence ABC is required triangle.

3. Construct a ΔKLM, for the following assumptions:

(i) $m\overline{KL} = 4.8cm$, $m\angle K = 45^\circ$ and $m\angle M = 60^\circ$

Sol: As the line segment $\overline{KL}$ is given, so firstly we will find $m\angle L$.

$\because m\angle K + m\angle M + m\angle L = 180^\circ$

$\Rightarrow 45^\circ + 60^\circ + m\angle L = 180^\circ \Rightarrow m\angle L = 180^\circ - 45^\circ - 60^\circ$

$= 75^\circ$

(208)

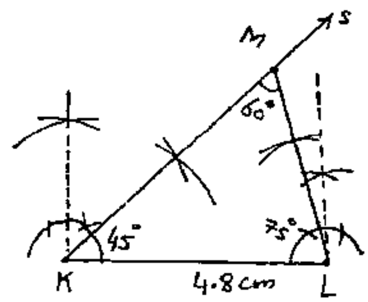
Steps of Constructions:

(i) Draw a line segment $\overline{KL}$ of measure 4.8cm.

(ii) With centre K, construct an angle of measure 45°.

(iii) Construct an angle 75° with centre L, which meets $\overrightarrow{KS}$ at M.

Therefore, KLM is the required triangle.



II. $m\overline{LM} = 3.8cm$, $m\angle N = 30°$ and $m\angle M = 75°$

Sol:- Steps of Constructions:

(i) Since line segment $\overline{LM}$ is given, so firstly we will find $m\angle L$.

$\because m\angle L + m\angle N + m\angle M = 180°$

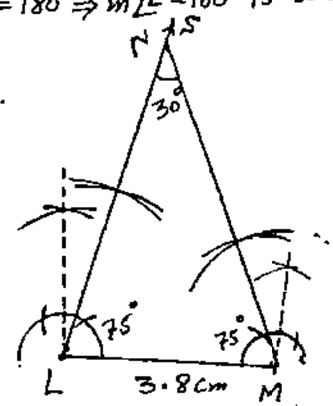
$\Rightarrow m\angle L + 30° + 75° = 180° \Rightarrow m\angle L = 180 - 75° - 30° = 75°$

(ii) Draw a line segment $\overline{LM}$ of measure 3.8cm.

(iii) With centre L, Construct angle of measure of 75°.

(iv) With centre M, construct an angle of measure 75° which meets $\overrightarrow{LS}$ at N.

So LMN is the required triangle.



III. $m\overline{KM} = 5cm$, $m\angle K = 105°$ and $m\angle N = 45°$

(209)

Sol: Steps of constructions:

(i) The line segment $\overline{KM}$ is given, so will find $\angle M$ firstly.

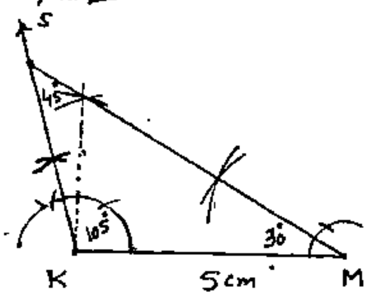
$\therefore m\angle K + m\angle N + m\angle M = 180°$

$\Rightarrow 105° + 45° + m\angle M = 180° \Rightarrow m\angle M = 180° - 150° = 30°$

(ii) Draw a line segment $\overline{KM}$ of length 5cm

(iii) With centre K, construct angle of measure 105°.

(iv) With centre M, construct an angle of measure 30° which meets $\overrightarrow{KS}$ at N. So KMN is the required triangle.



4. Construct a ΔABC (Whenever possible), for the following assumptions:

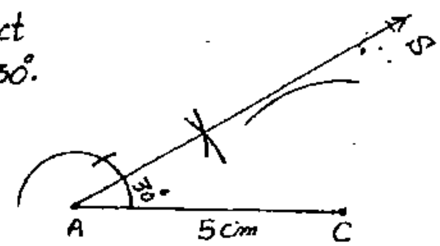
(i) $m\angle A = 30°$, $m\overline{BC} = 2.5$cm and $m\overline{AC} = 5$cm

Sol: Steps of constructions:

(i) Draw a line segment $\overline{AC}$ of measure 5cm.

(ii) With centre A, construct an angle of measure 30°.

(iii) With centre C, draw an arc length of measure 2.5cm. Which does not touches $\overrightarrow{AR}$ at any point. Therefore, ΔABC is not possible with given data.



ii. $m\angle B = 30^\circ$, $m\overline{AB} = 8cm$ and $m\overline{AC} = 4cm$.

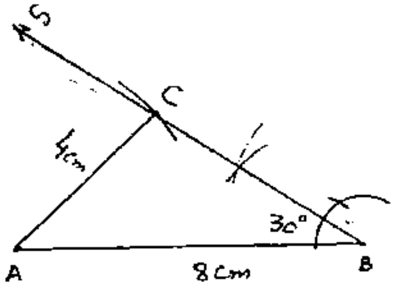
Sol: Steps of Constructions:

(i) Draw a line segment $\overline{AB}$ of length 8cm.

(ii) With centre B, construct an angle of measure of 30°

(iii) With centre A, draw an arc of length 4cm, which cuts $\overrightarrow{BS}$ at C. Join A to C.

ABC is the required triangle.



iii. $m\overline{AB} = 6cm$, $m\overline{AC} = 5.6cm$ and $m\angle B = 60^\circ$

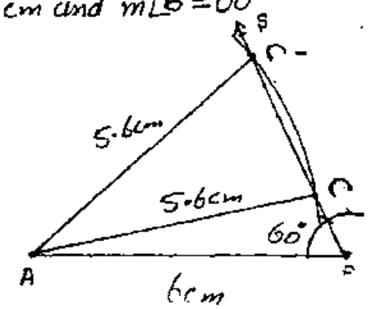
Sol: Steps of Constructions:

(i) Draw a line segment $\overline{AB}$ of length 6cm

(ii) Construct angle of measure 60° with centre B.

(iii) With cent... draw an arc of arc length 5.6 cm which meets at C and C' on $\overrightarrow{BS}$

(iv) Join C and C' to A. So ΔABC and $\Delta ABC'$ are formed with given data



IV. $m\overline{AB} = 7cm$, $m\overline{AC} = 5.5cm$, $m\angle B = 45^\circ$

Steps of Constructions:

(i) Draw a line segment of $m\overline{AB} = 6cm$ (ii) With centre B construct an angle of measure 45°.

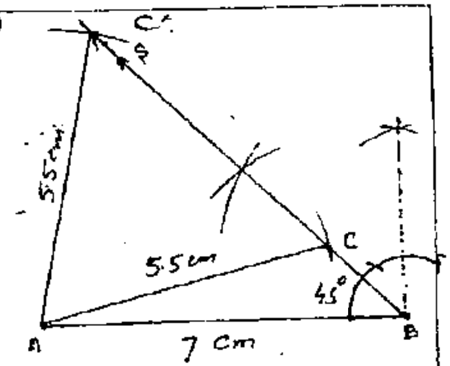
(iii) With centre A, draw an arc of length 5.6cm which

(211)

cuts $\overrightarrow{BS}$ at C and C'

(iv) Join C, C' to A

So ABC and ABC' are two possible triangles.



(V) $m\overline{AC} = 4.5$ cm, $m\overline{BC} = 7.5$ cm, $m\angle B = 45°$

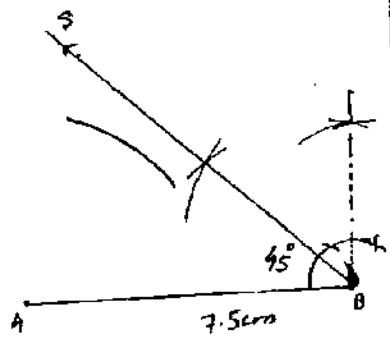
Soln.

(i) Draw $m\overline{AB} = 7.5$ cm

(ii) With centre B, construct angle of 45°

(iii) With centre A, draw and arc of length 4.5cm which does not touch $\overrightarrow{BS}$

So $\triangle ABC$ is not possible.



(VI) $m\overline{AB} = 6$cm, $m\overline{AC} = 2.5$ cm, $m\angle A = 60°$

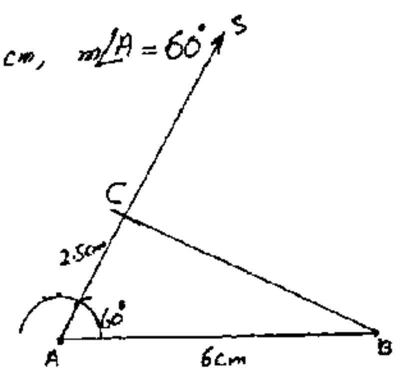
(i) Draw $m\overline{AB} = 6$cm

(ii) With centre A, construct angle of measure 60°

(iii) With centre A, draw an arc of length 2.5cm which cuts $\overrightarrow{AS}$ at C.

(iv) Join A to C.

ABC is the required triangle.



Angle Bisector:- A line which bised the angle into two equal parts is called angle bisector. Noted that the angle bisectors of a triangle are concurrent.

Right bisector: The line which is perpendicular to a line-segment and divides into two equal parts is called right bisectors. Noted that right bisectors in a triangle are concurrent.

Median: The line segment joining the mid point of the side of triangle to opposite vertex is called median. The medians of a triangle are concurrent.

Altitude: A ray drawn from the vertex of a triangle on opposite side which is also perpendicular is called altitude. All the altitudes of a triangle are concurrent.

Ex: 8.2

1. Construct a ΔABC and draw their angle bisectors, for the following assumptions:

(i) $m\overline{AB} = 3cm$, $m\overline{BC} = 4.5cm$, $m\overline{AC} = 4cm$.

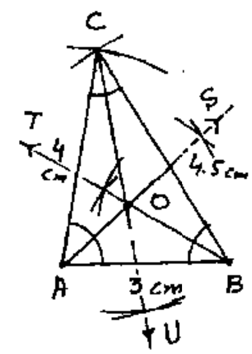
<u>Steps of construction:</u>

(i) Draw a line segment $\overline{AB}$ of length 3cm.

(ii) With centre A draw an arc of length 4cm.

(iii) With centre B draw another of length 4.5 cm which cutts the first arc at C. Join A and B to C.

(iv) Draw the bisectors $\overrightarrow{AS}$, $\overrightarrow{BT}$ and $\overrightarrow{CU}$ of the angles $\angle A, \angle B, \angle C$ respectively.



As the bisectors $\overrightarrow{AS}, \overrightarrow{BT}, \overrightarrow{CU}$ meets at 'O' so they are concurrent.

(213)

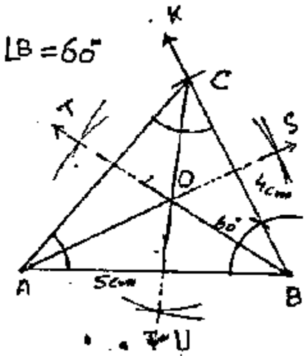
(ii) $m\overline{AB} = 5cm$, $m\overline{BC} = 4cm$, $m\angle B = 60^\circ$

Steps of construction

(i) Draw $m\overline{AB} = 5cm$

(ii) Construct angle of measure 60° with centre B.

(iii) With centre B draw an arc of length 4cm, which cuts $\overrightarrow{BK}$ at C. Join C to A



So ABC is the required triangle.

Draw the bisectors $\overrightarrow{AS}$, $\overrightarrow{BT}$ and $\overrightarrow{CU}$ of angles $\angle A$, $\angle B$ and $\angle C$ respectively. They meets at 'O' so all the angle bisectors are concurrent.

(iii) $m\overline{AB} = 5.5cm$, $m\overline{BC} = 5cm$, $m\overline{AC} = 6cm$

Steps of construction

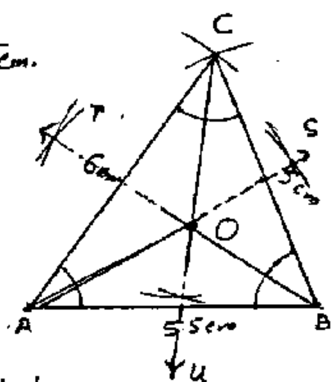
(i) Draw a line segment $m\overline{AB} = 5.5cm$.

(ii) With centre A, draw an arc of length 6cm.

(iii) With centre B, draw another arc of length 5cm which intersect the first arc at C.

(iv) Join C to A and B.

(v) Draw the bisectors $\overrightarrow{AS}$, $\overrightarrow{BT}$ and $\overrightarrow{CU}$ of $\angle A$, $\angle B$, $\angle C$ respectively.

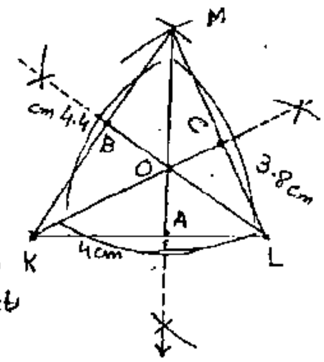


All the bisectors cuts each other at O so they are concurrent.

2. Construct $\triangle KLM$ and draw their altitudes for the following assumptions. Also verify the concurrency of these altitudes.

(i) $m\overline{KL} = 4cm$, $m\overline{LM} = 3.8cm$, $m\overline{KM} = 4.4cm$

(214)



Steps of construction

(i) Draw a line segment $\overline{KL}$ of length 4cm.

(ii) Draw an arc of length 4.4cm with centre K.

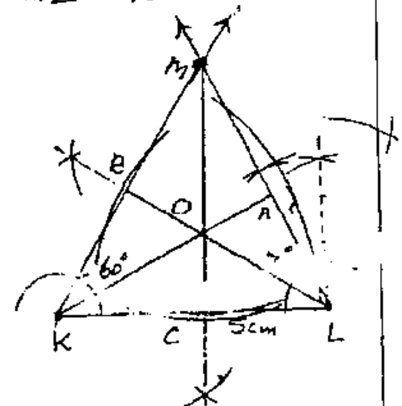
(iii) Draw another arc of length 3.8cm with centre L. Both arcs intersect at M.

(iv) Join M to K and L.

(v) Draw perpendiculars from K on $\overline{LM}$, L on $\overline{KM}$ and from M on $\overline{KL}$. $\overline{MA}$, $\overline{KC}$ and $\overline{LB}$ are the required altitudes which are concurrent.

ii. $m\overline{KL} = 5cm$, $m\angle K = 60^\circ$; $m\angle L = 75^\circ$

Steps of construction:



(i) Construct $\triangle KLM$ with given data

(ii) Draw perpendiculars from K, L and M on $\overline{KL}$, $\overline{KM}$, $\overline{LM}$

(iii) $\overline{AK}$, $\overline{CM}$ and $\overline{LB}$ are the required altitudes which are concurrent, as they passing through O.

iii. $m\overline{KL} = 4.8cm$, $m\angle K = 75^\circ$ and $m\overline{KM} = 5cm$.

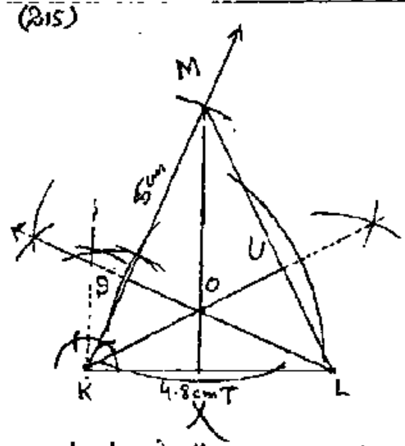
Steps of construction

(i) Construct $\triangle KLM$ with given data.

(ii) Draw perpendiculars $\overline{KA}$ from K on $\overline{LM}$, $\overline{LB}$ from L on $\overline{KM}$ and $\overline{MC}$ from M on $\overline{KL}$

(iii) $\overline{LB}$, $\overline{KA}$ and $\overline{KM}$ are the required altitudes.

As all the altitudes passing through O, so they are concurrent.



3. Construct $\triangle PQR$ and draw their perpendicular bisectors, for the following assumptions.

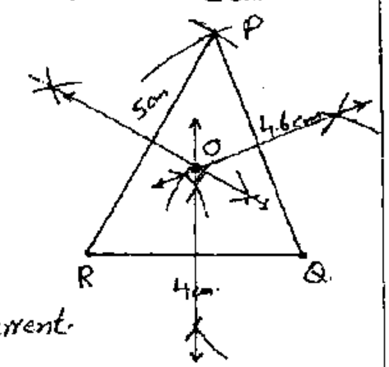
(i) $m\overline{PQ} = 4cm$, $m\overline{QR} = 4.6cm$ and $m\overline{PR} = 5cm$

Steps of construction:

(i) Construct $\triangle PQR$ with given data

(ii) Draw the right bisectors of the sides $\overline{QR}$, $\overline{PQ}$ and $\overline{PR}$.

(iii) All the bisectors meets at O. Therefore they are concurrent.



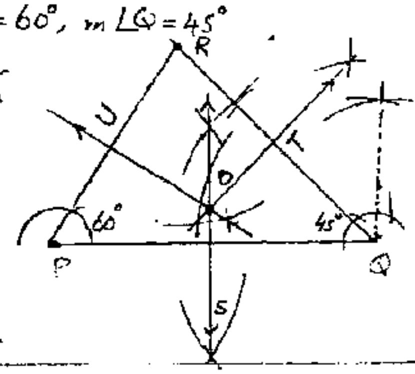
(ii) $m\overline{PQ} = 6cm$, $m\angle P = 60°$, $m\angle Q = 45°$

Steps of construction

(i) Construct $\triangle PQR$ with given data.

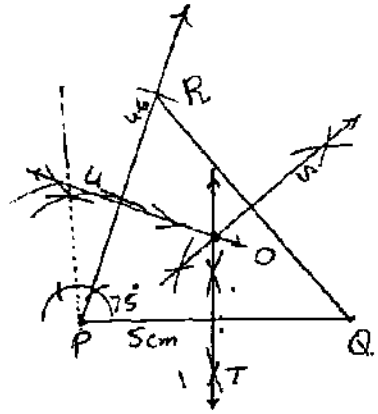
(ii) Draw the perpendicular bisectors of the sides.

(iii) They meets at O and concurrent.



iii. $m\overline{PQ} = 5cm$, $m\angle P = 75^\circ$, $m\overline{PR} = 4.6cm$.

Steps of Constructions:



(i) Construct $\triangle PQR$ with given data

(ii) Draw the perpendicular bisectors $\overrightarrow{OT}$, $\overrightarrow{OS}$ and $\overrightarrow{OU}$ of the sides $\overline{PQ}$, $\overline{QR}$, $\overline{PS}$ respectively.

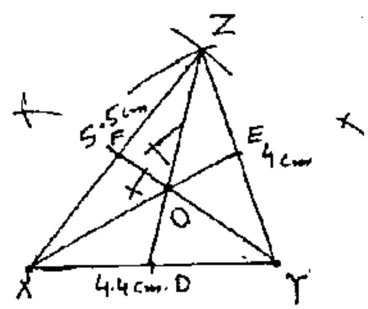
(iii) All the bisectors are concurrent.

4. Construct $\triangle XYZ$ and draw their medians for the following assumptions:

(i) $m\overline{XY} = 4.4cm$, $m\overline{YZ} = 4cm$ and $m\overline{XZ} = 5.5cm$

Steps of Construction

(i) Draw $\overline{XY}$ of length 4.4cm



(ii) Draw arcs of radii 5.5 cm and 4 cm with centres X and Y respectively which intersect each other at Z.

(iii) Join Z to X and Y to get $\triangle XYZ$

(iv) Find the mid points D, E, F of the sides $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$ respectively.

(V) Join the mid points with opposite vertices. $\overline{XE}$, $\overline{YF}$, $\overline{XE}$ are the required medians which are concurrent.

(Q17)

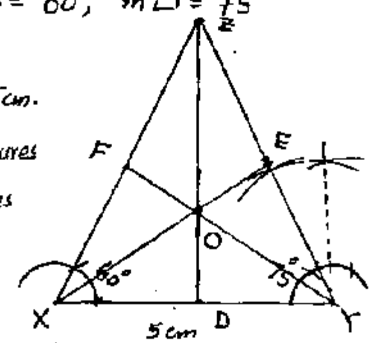
(ii) $m\overline{XY} = 5cm$, $m\angle X = 60°$, $m\angle Y = 75°$

Steps of construction

(i) Draw $\overline{XY}$ of length 5cm.

(ii) Construct angles of measures 60° and 75° with centres X and Y respectively. They intersects each other at Z.

So XYZ is required triangle.



(iii) Find the mid points D, E, F of sides $\overline{XY}$, $\overline{YZ}$, $\overline{XZ}$ respectively.

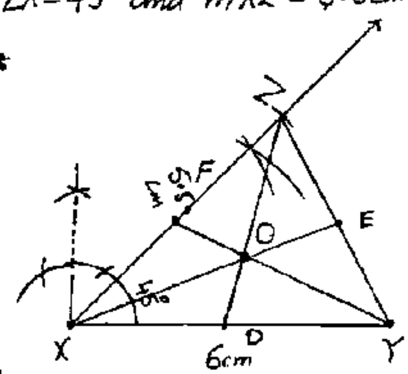
(iv) Join mid points to opposite vertices. $\overline{XE}$, $\overline{YF}$ and $\overline{XE}$ which are clearly concurrent.

(iii) $m\overline{XY} = 6cm$, $m\angle X = 45°$ and $m\overline{XZ} = 5.5cm$

Steps of constructions

(i) Construct $\triangle XYZ$ with given data

(ii) Find the mid points D, E and F of the sides $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$ respectively.



(iii) Join the mid points with opposite vertices.

$\overline{XE}$, $\overline{YF}$ and $\overline{ZD}$ are the required medians.

Since all the medians passing through O. So, the medians are concurrent.

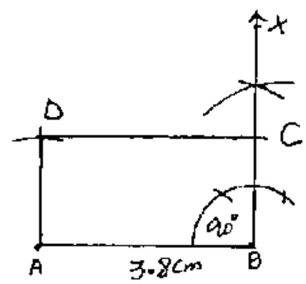
(218)

Exercise: 8.3

1. Construct a rectangle ABCD for the following data

(i) $m\overline{AB} = 3.8$ cm and $m\overline{BC} = 2$cm

Steps of Construction:



(i) Draw $\overline{AB}$ of measuring 3.8Cm.

(ii) At B draw $\overrightarrow{BX} \perp \overline{AB}$

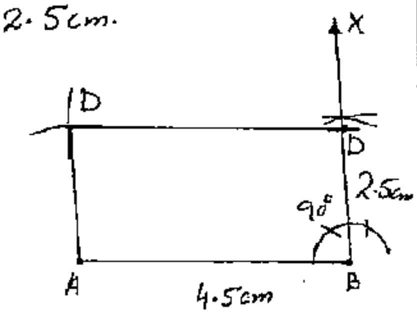
(iii) Cut off $\overline{BC}$, such that $m\overline{BC} = 3.5$cm

(iv) With centres A and C draw arcs with radii 2cm and 3.8cm respectively which meets each other at D.

(v) Join D to A and C. ABCD is the required rectangle.

(ii) $m\overline{AB} = 4.5$cm, $m\overline{AD} = 2.5$cm.

Steps of construction:



(i) Draw $\overline{AB}$ measuring 4.5cm

(ii) at B draw $\overrightarrow{BX} \perp \overline{AB}$

(iii) Cut off $\overline{BC}$, such that $mBC = 2.5$cm

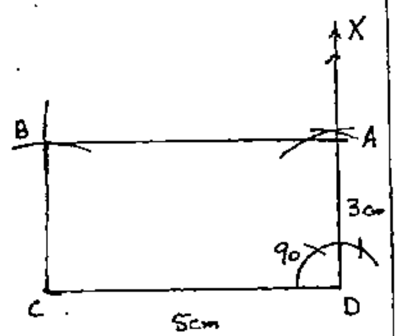
(iv) With centres A and C draw arcs of radii 2 Cm and 4.5 cm respectively which meets at D.

(v) Join D to A and C.

ABCD is the required rectangle.

(iii) $m\overline{BC} = 3cm$ and $m\overline{CD} = 5cm$

Steps of construction:



(i) Draw $\overline{CD}$ of measure 5cm.

(ii) Construct angle 90° at centre D.

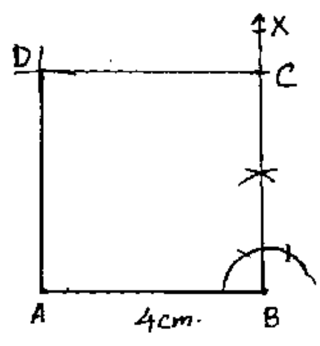
(iii). Cut off $\overline{AD}$ on $\overrightarrow{DX}$ s.t $m\overline{AD} = 3cm$

(iv) With centres A and C draw arcs of radii 5cm, 3cm respectively which intersects at B.

(v) Join B to C and A. ABCD is the required rectangle.

2. Construct a square ABCD with $m\overline{AB} = 4cm$ and $m\angle B = 90°$

Steps of construction:



(i) Draw a line segment $\overline{AB}$ of measure 4cm.

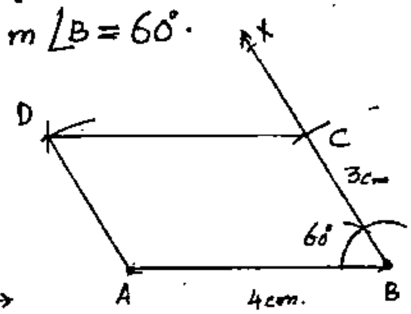
(ii) Construct $m\angle B = 90°$

(iii) Cut off $\overline{BC}$ from $\overrightarrow{BX}$ of length 4cm.

(iv) With centre A and C draw arcs of radii 4cm which intersects each other at D.

(v) Join D to C and A. ABCD is required square.

3. Construct a parallelogram ABCD with $m\overline{AB} = 4cm$ and $\overline{BC} = 3cm$, $m\angle B = 60°$.



Steps of construction

(i) Draw $\overline{AB}$ of measure 4cm.

(ii) Construct $m\angle B = 60°$

(iii) Cut off $\overline{BC}$ from $\overrightarrow{BX}$ such that $m\overline{BC} = 3cm$

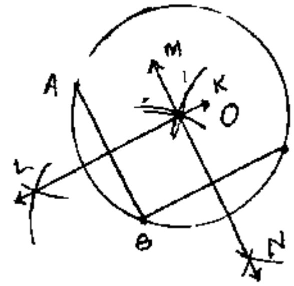
(iv) With centres C and A draw arcs of radii 4cm and 3 cm respectively, which intersect at D.

(v) Join D to C and A.

ABCD is the required parallelogram.

EXERCISE 8.4

1. Draw a circle with the help of round object. Locate its centre.



Steps of construction

(i) Draw a circle with round object

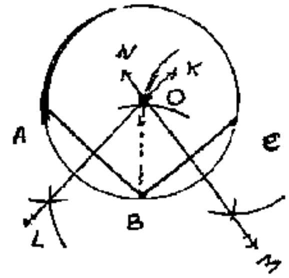
(ii) Take A, B, C on circumference of circle.

(iii) Join A and C to B.

(iv) Draw the right bisectors $\overleftrightarrow{KL}$ and $\overleftrightarrow{MN}$ of the sides $\overline{AB}$ and $\overline{BC}$ respectively. Which intersects at 'O'. O is the required centre.

2. Draw a circle which passes through non collinear points A, B, C.

Steps of construction:



(i) Take any three non-collinear points A, B, C.

(ii) Join A and C to B.

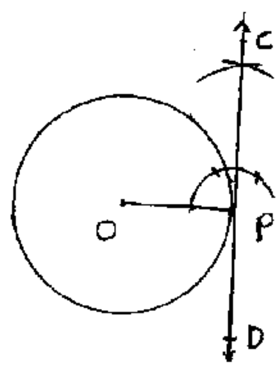
(iii) Draw the right bisectors $\overleftrightarrow{KL}$ and $\overleftrightarrow{MN}$ of the sides $\overline{AB}$ and $\overline{BC}$ respectively, which meets at O.

(iv) Join O to B.

(v) With centre 'O' draw a circle of radii equal to measure $\overline{OB}$. The circle passing through A, B and C.

3. Draw a circle of radius 2cm. Take a point on the circum-ference of the circle and draw a tangent to the circle at that point.

Steps of Construction:



(i) Draw a circle of radius 2cm with centre O.

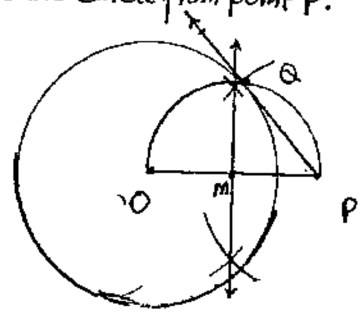
(ii) Take a point P on the circumference of circle.

(iii) Join O to P.

(iv) Draw $\overleftrightarrow{CD} \perp \overline{OP}$.
$\overleftrightarrow{CD}$ is the required tangent.

4. Draw a circle of radius 2.5cm. Take a point P which is at a distance 3cm from the centre of the circle. Draw a tangent to the circle from point P.

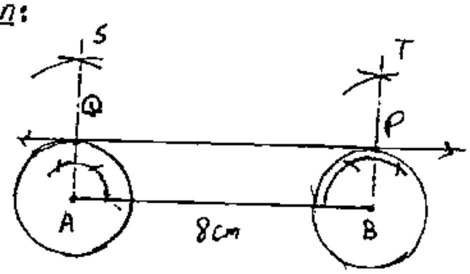
Steps of construction:



(i) Draw a circle with centre O of radii 2.5cm
(ii) Take a point P 3 cm distance from O.
(iii) Join O to P.
(iv) Draw the right bisector of $\overline{OP}$ which meets at M.
(v) With centre M draw a semi-circle of radii $\overline{OM}$ which intersect the circle at Q.
(vi) Join P to Q and extends. $\overrightarrow{PQ}$ is the required tangent.

5. Draw two circles of radii 2.5cm each, such that the distance b/w their centre is 8cm. Draw the direct common tangents to these circles.

Steps of construction:



(i) Draw $\overline{AB}$ of measure 8cm
(ii) Draw circles with centre A and B of radii 2.5cm
(iii) Draw $\overrightarrow{AS} \perp \overline{AB}$ and $\overrightarrow{BT} \perp \overline{AB}$. (iv) Q & P are the points of intersection. (v) Draw $\overleftrightarrow{PQ}$ which is the required tangent.

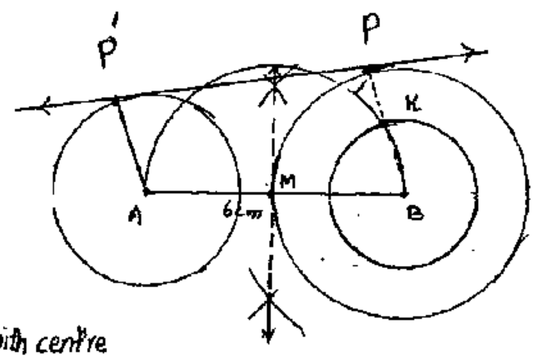
(223)

6. Centers of two circles of radii 2.5cm, 1.5cm are 6cm apart. Draw the direct common tangents to these circles.

Steps of construction:

(i) Draw $\overline{AB}$ of measure 6cm.

(ii) Draw circles with centers A and B of radii 1.5cm and 2.5cm respectively.



(iii) Draw a circle with centre B of radii = 2.5 − 1.5 = 1cm

(iv) Draw the right bisector of $\overline{AB}$

(v) Draw a semi-circle with center M of radii = $m\overline{AM}$ which intersect the inner circle at K. Extends K to P

(vi) Draw $\overline{AP'} \parallel \overline{BP}$ (vii) Draw $\overleftrightarrow{PP'}$ which is the required tangents.

7. Two circles have the same radius of 2cm each. Their centers are 6cm a part. Draw the transverse common tangents.

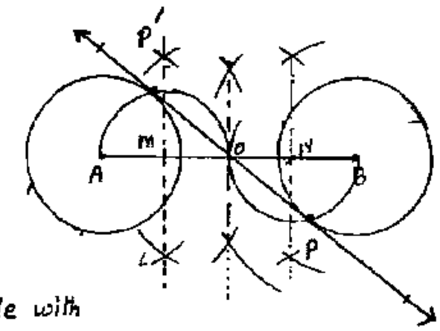
Steps of construction:

(i) Draw $\overline{AB}$ of measure 6cm

(ii) Draw circles with center A and of radii 2cm.

(iii) Divide $\overline{AB}$ into four equal parts i.e $\overline{AM}$, $\overline{MO}$, $\overline{ON}$ and $\overline{NB}$.

(iv) Draw a semi circle with centre M which intersects the circle at P'.



(v) Draw a semi circle with centre N, radii $\overline{ON}$ in opposite side of 1st semi-circle which intersect the circle at P.

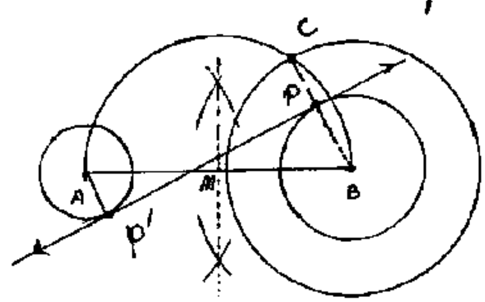
(vi) Join PP' and draw $\overleftrightarrow{PP'}$, which is the required.

8. Draw transverse common tangent to two circles having radii 2.5 and 1.5cm and their centres are 7.5cm apart.

Steps of Construction

(i) Draw $\overline{AB} = 7.5cm$

(ii) Draw circles at centre A and B with radii 1.5cm and 2.5cm.



(iii) Bisect $\overline{AB}$ at M. (iv) Draw a semi circle with centre M and radii = $\overline{AM}$

(iv) Draw a circle with centre B and radii = 1.5+2.4 = 4cm

(v) Draw a semi circle with centre M, cuts the circle at C. Join C to B, which intersects the inner circle at P. (vi) Draw $\overleftrightarrow{AP'} \parallel \overline{BP}$.

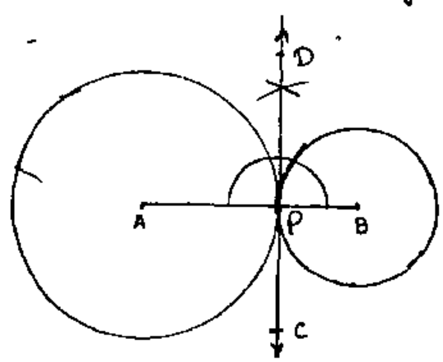
(vi) Draw $\overleftrightarrow{PP'}$ which is the required tangent.

9. Draw a tangent to the circles of radii 1.5cm and 2.5 cm that are touching each other externally.

Steps of construction:

(i) Draw $\overline{AB}$ of measure 4 cm.

(ii) Draw circles with centres A and of radii 2.5cm and 1.5cm which intersects at P.



(iii) Draw $\overleftrightarrow{CD} \perp \overline{AB}$ at point P. $\overleftrightarrow{CD}$ is the required tangent.

Revision Exercise 8

1. Which of the following statements are true and which are false.

(i) The point of intersection of the altitudes of any triangle is equidistant from its vertices. (F)

(ii) If measures of three line segments are given, it is not necessary that a triangle be constructed from them. (T)

(iii) If measure of two angles and one side are given, we can draw a triangle. (T)

(iv) Only one tangent can be drawn to a circle from a point which lies on its circumference. (F)

(226)

2. Fill in the blanks.

(i) Line segments joining the vertex of a triangle to the midpoint of its opposite side is called a Median

(ii) A line which passes through the midpoint of a line Segment and is also perpendicular to it, is called its Right Bisector

(iii) Vertices of a triangle are equidistant from the point of intersection of its perpendicular bisector.

(iv) The point of intersection of bisectors of three angles of a triangle is equidistant from its Sides.

(v) Perpendicular drawn from vertex of a triangle to its opposite side is called Altitudes

(vi) If three or more lines passing through same point The lines are said to be concurrent

vii The point of intersection of the medians of a triangle divides 2:1 ratio.

viii Bisectors of angles of a triangle are Concurrent.

3. Choose the correct option.

(i) a (ii) b (iii) a (iv) a (v) d

(vi) a (vii) b

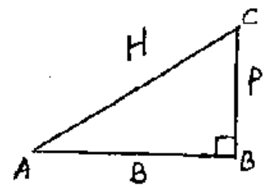
End of chap # 8

Unit 9

AREAS AND VOLUME

Pythagorus Theorem :- In a right angled triangle square of the hypotenuse is equal to the sum of squares of base and altitude i.e

$$H^2 = B^2 + P^2$$



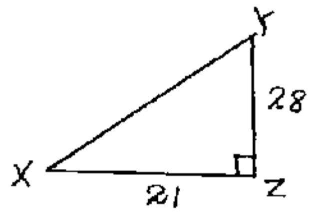
EXERCISE 9.1

1. In ΔXYZ, $m\angle Z = 90°$.

Fill in the blanks in the following table :

Soln:-

In ΔXYZ



$m\overline{YZ} = 28$, $m\overline{XZ} = 21$

$m\overline{XY} = ?$

By Pythagorus Theorem;

$$(m\overline{XY})^2 = (m\overline{XZ})^2 + (m\overline{YZ})^2$$

$$= (21)^2 + (28)^2$$

$$= 784 + 441$$

$$= 1225$$

Taking the square root on both sides we get.

$$\sqrt{(m\overline{XY})^2} = \sqrt{1225} = \sqrt{35 \times 35} = \sqrt{35^2}$$

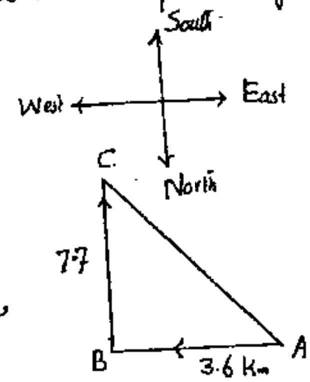
$$\Rightarrow m\overline{XY} = 35$$

Similarly by using Pythagorus Theorem the remaining parts can be find and the complete table is below.

(228)

	$m\overline{XY}$	$m\overline{YZ}$	$m\overline{XZ}$
(i)	35	28	21
(ii)	5.2	2	4.8
(iii)	6.5	5.2	3.9
(iv)	37.5	36	10.5
(v)	9.7	6.5	7.2
(vi)	22.5	21.6	6.3

2. Asad travelled a distance of 3.6 Kms on his bycycle towords West. He then travelled distance of 7.7 Kms towards South. Find the total distance he coverd from starting point.



Soln:- In the given fig.

Total distance $= m\overline{AC} = ?$

$m\overline{AB} = 3.6$ Km

$m\overline{BC} = 7.7$ Km.

Using Pythagorus theorem on right angle triangle ABC we have,

$(m\overline{AC})^2 = (m\overline{AB})^2 + (m\overline{BC})^2$

$= (3.6)^2 + (7.7)^2$

$= 12.96 + 59.29 = 72.25$

Taking the square root on both sides

$\sqrt{(m\overline{AC})^2} = \sqrt{72.25} \Rightarrow m\overline{AC} = 8.5$

$\therefore$ Asad travelled distance 8.5 Km from starting point.

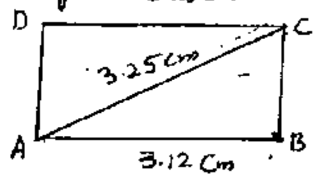
(229)

3. Diagonal of a rectangle is 3.25 cm long. Find the width of the rectangle of its length is 3.12 cm.

Soln: In the rectangle ABCD,

ΔABC is right angle.

So by Pythagorus Theorem



$(m\overline{AC})^2 = (m\overline{AB})^2 + (m\overline{BC})^2$

$(3.25)^2 = (3.12)^2 + (m\overline{BC})^2$

$10.5625 = 9.7344 + (m\overline{BC})^2$

$m\overline{AC} = 3.25\,cm$

$m\overline{AB} = 3.12\,cm$

$m\overline{BC} = ?$

$\Rightarrow (m\overline{BC})^2 = 10.5625 - 9.7344 = 0.8281$

Taking Square-root on both sides we get

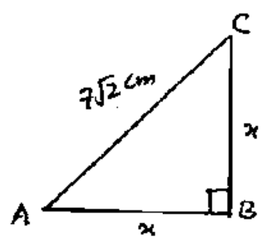
$\sqrt{(m\overline{BC})^2} = \sqrt{0.8281} \Rightarrow m\overline{BC} = 0.91\,cm$

$\therefore$ Width of rectangle $= 0.91\,cm$

4. Hypotenuse of an isoceles right-angled triangle is $7\sqrt{2}$ cm. Find the length of congruent sides of the triangle.

Soln: Suppose side of Isoceles triangle $= x$ cm

Then applying Pythagorus theorem to ΔABC,



$(m\overline{AC})^2 = (m\overline{AB})^2 + (m\overline{BC})^2$

$(7\sqrt{2})^2 = x^2 + x^2$

$49 \times 2 = 2x^2$

$\Rightarrow 2x^2 = 98$

$\Rightarrow x^2 = \frac{98}{2} = 49$

$\Rightarrow \sqrt{x^2} = \sqrt{49} \Rightarrow x = 7$

$\therefore$ Length of congruent sides $= 7$ cm

(230)

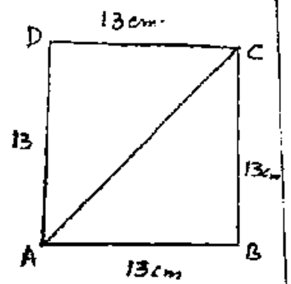
5. Side of a square is 13 cm long. Find the length of its diagonal.

Soln:- Let ABCD be a square.

Length of each of its side is 13cm.

Let length of hyp = x cm

Then in $\triangle ABC$



$(m\overline{AC})^2 = (m\overline{AB})^2 + (m\overline{BC})^2$

$\Rightarrow x^2 = (13)^2 + (13)^2$

$\Rightarrow x^2 = 169 + 169 = 338$

Taking square on both sides.

$\sqrt{x^2} = \sqrt{338} = \sqrt{169 \times 2} = \sqrt{169} \times \sqrt{2}$

$\Rightarrow x = 13\sqrt{2}$

$\therefore$ Length of diagonal $\overline{AC} = 13\sqrt{2}$ cm

6. A ladder 13m long rests against a wall and reaches a hight of 12m on the wall. Find the distance from the foot of the ladder to the wall.

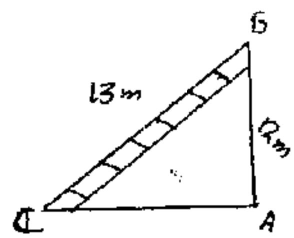
S: In the given fig.

$\overline{AB}$ denoted hight of wall

$\overline{AC}$ is distance of foot from wall.

$\overline{BC}$ denotes length of ladder

So in right angle triangle ABC



$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2$

$\Rightarrow (13)^2 = (m\overline{AC})^2 + (12)^2$

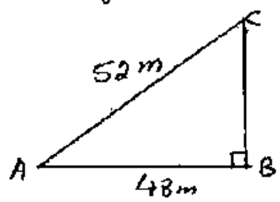
$\Rightarrow 169 = (m\overline{AC})^2 + 144 \Rightarrow (m\overline{AC})^2 = 169 - 144 = 25$

$\Rightarrow \sqrt{(m\overline{AC})^2} = \sqrt{25} \Rightarrow m\overline{AC} = 5m$

Hence distance of the foot of ladder from wall is 5m.

7. A right triangular shaped piece of land has a hypotenuse of 52m. Find its altitude if its base is 48m.

Soln.



Since $\triangle ABC$ is right angled

So by Pythagoras Theorem

$(m\overline{AC})^2 = (m\overline{AB})^2 + (m\overline{BC})^2$ —(i)

But $m\overline{AC} = 52$ m, $m\overline{AB} = 48$ m, $m\overline{BC} = ?$

Putting these values in Eq (i) we get

$(52)^2 = (48)^2 + (m\overline{BC})^2$

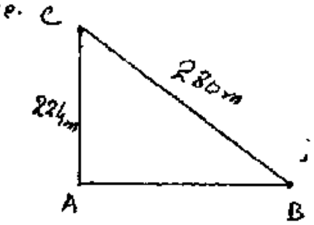
$\Rightarrow (m\overline{BC})^2 = (52)^2 - (48)^2 = 2704 - 2304 = 400$

Taking Square root on both sides.

$\sqrt{(m\overline{BC})^2} = \sqrt{400} = \sqrt{20^2} \Rightarrow m\overline{BC} = 20m$

$\therefore$ Altitude of triangular shaped region = 20 m

8. Top of a light house is 224m from the surface of the water and 280m from a boat sailing in the sea. Find the distance b/w boat and light house.



Soln: Let B be the position of boat and C be the position of light house.

Then from $\triangle ABC$,

$(m\overline{BC})^2 = (m\overline{AB})^2 + (m\overline{AC})^2$

$(280)^2 = (m\overline{AB})^2 + (224)^2$

$m\overline{AB} = ?$
$m\overline{BC} = 280m$
$m\overline{AC} = 224m$

$\Rightarrow 78400 = (m\overline{AB})^2 + 50176$

$\Rightarrow (m\overline{AB})^2 = 78400 - 50176 = 28224$

$\Rightarrow (m\overline{AB})^2 = 168 \times 168 \Rightarrow \sqrt{(m\overline{AB})^2} = \sqrt{(168)^2}$

$\Rightarrow m\overline{AB} = 168m$

So distance of boat from light house = 168 m.

Area of triangle:-

Case1:- If three sides of a triangle are given then Hero's formula is used for area. The formula is

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $S = \frac{a+b+c}{2}$

Exple: Find the area of triangular region ABC, where measure of sides are: $a=4cm$, $b=5cm$, $C=7cm$

Soln: Here $a=4$, $b=5$ and $C=7$

and $S = \frac{a+b+c}{2} = \frac{4+5+7}{2} = \frac{16}{2} = 8$

According to Hero's formula:

$$Area = \sqrt{S(s-a)(s-b)(s-c)}$$

$$= \sqrt{8(8-4)(8-5)(8-7)} = \sqrt{8 \times 4 \times 3 \times 1}$$

$$= \sqrt{8 \times 12} = \sqrt{2 \times 4 \times 3 \times 4} = 4\sqrt{6}\ cm^2$$

CaseII When the measure of the base and altitude is give then the formula for area of triangle is:

$$\text{Area of triangle} = \frac{1}{2}(\text{Base} \times \text{Altitude})$$

Exp: Area of triangular region is $364\ m^2$. Find the base when its altitude is $52m$.

Soln:- Area of triangle $= 364\ m^2$

Let base $= x\ m$

Altitude $= 52\ m$

As Area of triangle $= \frac{1}{2}(\text{base} \times \text{altitude})$

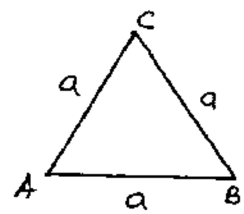
$$364 = \tfrac{1}{2}(x \times 52)$$

$$\Rightarrow \frac{728}{52} = \frac{x \times 52}{52} \Rightarrow x = 14m.$$

Area of Equilateral triangle

Let ABC be equilateral triangle each sides is of length a. i.e $a=a$, $b=a$, $c=a$ then

$S = \frac{a+b+c}{2} = \frac{a+a+a}{2} = \frac{3a}{2}$



Area of $\triangle ABC = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$

$= \sqrt{\frac{3a}{2}\left(\frac{3a-2a}{2}\right)\left(\frac{3a-2a}{2}\right)\left(\frac{3a-2a}{2}\right)}$

$= \sqrt{\frac{3a}{2}\cdot\frac{a}{2}\cdot\frac{a}{2}\cdot\frac{a}{2}} = \sqrt{\frac{3a^4}{16}} = \frac{a^2\sqrt{3}}{4}.$

EXERCISE : 9.2

1. Find the area of triangular region whose sides are:

(i) 21 cm, 17cm, 10 cm

Soln:- Here $a = 21cm$, $b = 17cm$, $c = 10cm$

As $S = \frac{a+b+c}{2}$

$\therefore S = \frac{21+17+10}{2} = \frac{48}{2} = 24$

According to Hero's Formula:

Area of Triangle $= \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{24(24-21)(24-17)(24-10)}$

$= \sqrt{24 \times 3 \times 7 \times 14}$

$= \sqrt{4 \times 6 \times 3 \times 7 \times 2 \times 7}$

$= \sqrt{4 \times 3 \times 3 \times 2 \times 2 \times 7 \times 7}$

$= \sqrt{4^2 \times 3^2 \times 7^2} = 4 \times 3 \times 7$

$= 84\ cm^2$

(ii) 20m, 13m, 11m

Soln: Here $a = 20\ m$, $b = 13\ m$, $c = 11\ m$.

As $S = \frac{a+b+c}{2}$

$\therefore S = \frac{20+13+11}{2} = \frac{44}{2} = 22$

According to Hero's formula:

$$\text{Area of triangle} = \sqrt{S(S-a)(S-b)(S-c)} \quad \text{—(1)}$$

Putting the values in Eq (1)

$$\text{Area of triangle} = \sqrt{22(22-20)(22-13)(22-11)}$$

$$= \sqrt{22 \times 2 \times 9 \times 11}$$

$$= \sqrt{11 \times 2 \times 2 \times 9 \times 11} = \sqrt{11^2 \times 2^2 \times 3^2}$$

$$= 11 \times 2 \times 3 = 66\ m^2$$

(iii) 13m, 14m, 15m

Soln:- Here $a = 13m$, $b = 14m$, $c = 15m$

$\because S = \frac{a+b+c}{2}$

$\therefore S = \frac{13+14+15}{2} = \frac{42}{2} = 21$

According to Hero's Formula:

$$\text{Area of Triangle} = \sqrt{S(S-a)(S-b)(S-c)}$$

Putting the values

$$\text{Area of triangle} = \sqrt{21(21-13)(21-14)(21-15)}$$

$$= \sqrt{21 \times 8 \times 7 \times 6}$$

$$= \sqrt{3 \times 7 \times 2 \times 4 \times 7 \times 2 \times 3}$$

$$= \sqrt{3^2 \times 7^2 \times 4^2}$$

$$= 3 \times 7 \times 4$$

$$= 84\ m^2$$

(235)

(iv) 56 cm, 39cm, 25cm

Soln:- Here $a = 56cm$, $b = 39cm$, $c = 25cm$

As $s = \frac{a+b+c}{2}$

$\therefore s = \frac{56+39+25}{2}$ ∵ by putting the values

$\Rightarrow s = \frac{120}{2} = 60$

According to Hero's formula:

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60(60-56)(60-39)(60-25)}$

$= \sqrt{60 \times 4 \times 21 \times 35}$

$= \sqrt{5 \times 3 \times 4 \times 4 \times 3 \times 7 \times 5 \times 7}$

$= \sqrt{5^2 \times 3^2 \times 4^2 \times 7^2}$

$= 5 \times 3 \times 4 \times 7 = 420\ cm^2$

2. Find the area of the triangles, for the assumptions:

(i) base = 16cm, altitude = 7cm

Soln:- Area of triangle $= \frac{1}{2} \cdot$ base $\cdot$ altitude

$= \frac{1}{2} \times 16 \times 7$

$= 8 \times 7$

$= 56\ cm^2$

(ii) base = 5m, altitude = 12m

Soln: Area of triangle $= \frac{1}{2} \times$ Base $\times$ Altitude

$= \frac{1}{2} \times 5 \times 12$

$= 5 \times 6$

$= 30\ m^2$

(iii) base = 3.4m, altitude = 5.5m

Soln: Area of triangle $= \frac{1}{2} \times$ base $\times$ altitude

$= \frac{1}{2} \times 3.4 \times 5.5$

$= 1.7 \times 5.5$

$= 9.35\ m^2$

(iv) base $= 3\frac{1}{7}$ m, altitude $= 8\frac{3}{4}$ m

Soln: Area $= \frac{1}{2} \times$ base $\times$ altitude

$= \frac{1}{2} \times 3\frac{1}{7} \times 8\frac{3}{4}$

$= \frac{1}{2} \times \frac{22}{7} \times \frac{35}{4}$

$= \frac{11 \times 5}{4} = \frac{55}{4}\ m^2$

3. Area of triangular region is $72m^2$. Find the altitude if base is 9m.

Soln:- Given that;

Area of triangular region = $72m^2$

Base = 9m

Let its altitude = x m.

As, Area of a triangle = $\frac{1}{2}$ × base × altitude

$\Rightarrow 72 = \frac{1}{2} \times 9 \times x$ ∵ Putting the values

$72 = \frac{9}{2} \times x$

$\frac{2}{9} \times 72 = \frac{2}{9} \cdot \frac{9}{2} \cdot x$ multiplying b.s by $\frac{2}{9}$

$16 = x \Rightarrow x = 16m$

So Altitude = 16m

4. Area of a triangular region is $9.9m^2$. Find its base when the altitude is of 5.5m.

Soln:- Given that; Area of triangular region = $9.9m^2$

Let base = x m

Altitude = 5.5m

As area of triangular region = $\frac{1}{2}$ · base · altitude

$\Rightarrow 9.9 = \frac{1}{2} \times x \times 5.5$

$\Rightarrow 19.8 = x \times 5.5$ multiplying b.s by 2.

$\Rightarrow \frac{19.8}{5.5} = \frac{x \times 5.5}{5.5} \Rightarrow x = 3.6m$

∴ Base of the triangle = 3.6m

5. Find the area of equilateral triangles if length of one side is

(i) 4cm.

Soln:- Here $a = 4cm$

As Area of equilateral triangle = $\frac{\sqrt{3}}{4}a^2$ ——(i)

Putting the values in Eq (i)

$\therefore$ Area of triangle $= \frac{\sqrt{3}}{4}(4)^2$

$= \frac{\sqrt{3}}{4} \times 16 = 4\sqrt{3}\ cm^2$

(ii) $a = 5\sqrt{2}$ cm.

Soln:- Area of triangle $= \frac{\sqrt{3}}{4}a^2$ ——— (i)

Putting the value of 'a' in Eq (i) we get.

Area of triangle $= \frac{\sqrt{3}}{4}(5\sqrt{2})^2$

$= \frac{\sqrt{3}}{4}(5)^2(\sqrt{2})^2$

$= \frac{\sqrt{3}}{4} \times 25 \times 2 = \frac{25\sqrt{3}}{2}\ cm^2$

6. Find the length of the side of triangle when area is:

(i) $9\sqrt{3}\ cm^2$

Soln:- Here area of triangle $= 9\sqrt{3}\ cm^2$

Side of triangle $= a = ?$

As area of equilateral triangle is:

Area $= \frac{\sqrt{3}}{4}a^2$ ——— (i)

Putting the value of area in Eq (i)

$9\sqrt{3} = \frac{\sqrt{3}}{4}a^2$

$\Rightarrow \frac{9}{1} = \frac{a^2}{4} \Rightarrow a^2 = 36$

Taking square root on both sides

$\sqrt{a^2} = \sqrt{36} \Rightarrow \boxed{a = 6cm}$

(ii) $25\sqrt{3}\ cm^2$

Soln:- Here area of triangle $= 25\sqrt{3}\ cm^2$

Let side of equilateral triangle $= a = ?$

As area of triangle is:

Area $= \frac{\sqrt{3}}{4}a^2$ ——— (i)

Putting the value of area in Eq (1)

$$25\sqrt{3} = \frac{\sqrt{3}}{4}a^2$$

$\Rightarrow \frac{25}{1} = \frac{a^2}{4}$ ÷ing b.s by $\sqrt{3}$

$\Rightarrow a^2 = 100$ ∵ by cross multiplication

$\Rightarrow \sqrt{a^2} = \sqrt{100} \Rightarrow a = 10$

∴ Side of equilateral triangle = 10cm

AREAS AND PERAMETER OF QUADRATERALS

(i) Area of rectangle = Length x Width

Perimeter of rectangle = 2(Length + Width).

(ii) Area of a square = Side x Side = $(Side)^2$

Perimeter of a square = 4 x Side.

(iii) Area of parallelogram = Base x Altitude.

Perimeter of parallelogram = 2(Length + Width).

EXERCISE 9.3

1. Find the area of rectangles having the following:

(i) Length = 18 m, breadth = 6.5m

Soln:- Here length = 18m
breadth = 6.5m

As, Area of rectangle = Length x breadth

∴ Area of rectangle = 18m x 6.5m = 117 m^2

(ii) Length = 19.6m, breadth = 9.5m

Soln:- Length = 19.6m, breadth = 9.5m

As area of rectangle = Length x breadth

(239)

∴ Area of rectangle = 19.6 x 9.5
= 186.2 m^2

(iii) Length = 27.5m , breadth = 17.8m

Soln:- Here, Length = 27.5m, breadth = 17.8m

∵ Area of rectangle = Length x breadth

∴ Area of rectangle = 27.5 x 17.8
= 489.5 m^2

2. Find the area of parallelogram with the following assumption:

(i) Base = 27cm , altitude = 18.5cm

Soln:- Here, base of parallelogram = 27cm, altitude = 18.5cm

∵ Area of parallelogram = base x altitude
= 27 x 18.5
= 499.5 cm^2

∴ Area of parallelogram = 499.5 cm^2

(ii) Base = 19.2 cm, altitude = 8.5cm

Soln:- As area of parallelogram = Base x altitude

∴ Area of parallelogram = 19.2cm x 8.5cm
= 163.2 cm^2

∴ The required area = 163.2 cm^2

(iii) Base = 46.5cm , altitude = 19.2cm.

Soln:- Here, base = 46.5cm, altitude = 19.2cm

∵ Area of parallelogram = base x altitude.

∴ Area of parallelogram = 46.5 x 19.2
= 892.8 cm^2

∴ The required area = 892.8 cm^2

(240)

3. Find the area of square having following sides.

(i) 17 cm

Soln:- Length of side = 17 cm

As area of Square = Side x Side

∴ Area of Square = $17 \times 17 = 289 cm^2$

∴ Required are = $289 cm^2$

(ii) 12.6 m

Soln:- Length of side = 12.6 m

As, area of square = Side x Side

∴ area of square = $12.6 \times 12.6 = 158.76 m^2$

∴ required area = $158.76 m^2$

(iii) 8.9 cm

Soln:- Length of side = 8.9 cm

area of square = side x side.

∴ area of square = $8.9 \times 8.9 = 79.21 cm^2$

∴ The required area = $79.21 cm^2$

4. Find the area of four walls of a room having the following assumptions:

(i) Length = 6.9 m, breadth = 4.7 m, hight = 4.5 m

Soln:- Given that: Length = 6.9 m
breadth = 4.7 m
hight = 4.5 m

Area of four walls = $2(l+b) \times h$

$= 2(6.9+4.7) \times 4.5$ putting the value

$= 2(11.6) \times 4.5$

$= 23.6 \times 4.5 = 104.4 m^2$

∴ The required area = $104.4 m^2$

(241)

(ii) Length = 5.4m, breadth = 4.6m, Hight = 3.5m

Soln Given that $l = 5.4m$, $b = 4.6m$, $h = 3.5m$

$\because$ Area of Four walls $= 2(l+b)\times h$

$= 2[5.4+4.6]\times 3.5$

$= 2(10)\times 3.5$

$= 20\times 3.5 = 70m^2$

$\therefore$ The required area $= 70m^2$

(iii) Length = 4.9 m, breadth = 4.1m, hight = 4.2m

Soln:- Given that: $l = 4.9m$, $b = 4.1m$, $h = 4.2m$

As Area of 4 walls $= 2(l+b)\times h$

$= 2(4.9+4.1)\times 4.2$ By putting the values

$= 2(9)\times 4.2$

$= 18\times 4.2 = 75.6\,m^2$

$\therefore$ The required area $= 75.6m^2$

5. Area of a rectangle is $315m^2$. If the adjacent sides are in the ratio 5:7. Then find out the length and breadth.

Soln:- Area of rectangle $= 315m^2$ —— (i)

Ratio b/w adjacent sides = 5:7

$\Rightarrow$ Area of rectangle $= l\times b = 5x\cdot 7x = 35x^2$ (ii)

comparing eq(i) and eq(ii) we get

$35x^2 = 315 \Rightarrow \frac{35x^2}{35} = \frac{315}{35} \Rightarrow x^2 = 9$

$\therefore \sqrt{x^2} = \sqrt{9} \Rightarrow \boxed{x = 3}$

$\therefore$ Length of rectangle $= 5x = 5\times 3 = 15m$

Breadth of rectangle $= 7x = 7\times 3 = 21m$

(242)

6. There is a ratio of 7:13 in b/w the sides of a rectangle. Find the measures of sides of its area is $1456m^2$.

Soln:- Given that area of rectangle = $1456\,m^2$ —— (i)

Ratio between there sides = 7 : 13

∴ Area of rectangle = $7x \cdot 13x = 91x^2$ —— (ii)

Comparing Eq (i) and Eq (ii) we get

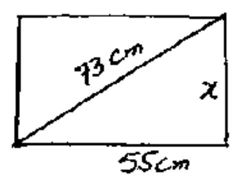
$$91x^2 = 1456 \Rightarrow \frac{91x^2}{91} = \frac{1456}{91}$$

$$\Rightarrow x^2 = 16 \Rightarrow \sqrt{x^2} = \sqrt{16} \Rightarrow \boxed{x=4}$$

∴ Length of rectangle $= 13x = 13 \times 4 = 52\,m$

Breadth of rectangle $= 7x = 7 \times 4 = 28\,m$

7. Length of rectangle is 55cm. If its diagonal is 73cm Find its area.



Soln:- Length of rectangle = 55cm

Lets its breadth = x cm

diagonal of rectangle = 76cm

According to Pythagorus Theorem;

$$H^2 = P^2 + B^2 \Rightarrow B^2 = H^2 - P^2 = (73)^2 - (55)^2$$

$$\Rightarrow B^2 = 5329 - 3025 \Rightarrow x^2 = 2304 \quad \because B = x$$

Taking Square root on both sides

$$\sqrt{x^2} = \sqrt{2304} = \sqrt{48 \times 48} \Rightarrow x = 48cm$$

∴ Area $= L \times B = 55 \times 48 = 2640cm^2$

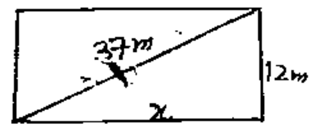
8. Width of a rectangle is 12m. Find its area, if the measure of its diagonal is 37m.

Soln:- Width of rectangle = 12m

Let Lenght of rectangle = x m

Length of diagonal = 37m

According to Pythagorus Theorem;



$(37)^2 = x^2 + (12)^2$

$\Rightarrow x^2 = (37)^2 - (12)^2 = 1369 - 144 = 1225$

Taking square root on both sides

$\sqrt{x^2} = \sqrt{1225} \Rightarrow x = \sqrt{35 \times 35} = 35m$

As Area of rectangle = L x W

$\therefore$ Area = $35 \times 12 = 420m^2$

9. Area of parallelogram is $13.5m^2$. Find its altitude, if its base is 9m.

Soln:- Given that area of llgram = $13.5m^2$

Base of parallelogram = 9m

Let Altitude = x m

$\because$ Area of parallelogram = Base x Altitude

$\therefore 13.5 = 9 \times x \Rightarrow 9x = 13.5$

$\frac{9x}{9} = \frac{13.5}{9}$ ÷ing by 9

$\Rightarrow x = 1.5m$

$\therefore$ Altitude of parallelogram = 1.5m.

10. Altitude of parallelogram is 6.5 cm. Find its base if the area of parallelogram $55.25cm^2$

Soln:- Area of parallelogram = $55.25cm^2$

Altitude of parallelogram = 6.5cm

Let base of it = x cm

$\because$ Area of parallelogram = Base x Altitude.

$\therefore$ Area of parallelogram $= 6.5 \times x$

$\Rightarrow \quad 6.5x = 55.25$

$\Rightarrow \quad x = \frac{55.25}{6.5} = 8.5$

So Length $= 8.5$ cm.

11. Area of a square is $272.25 m^2$. Find its perimeter.

Soln:- Let side of a square $= x$ m

$\therefore$ Area of square $= x \cdot x = x^2$

But given that area $= 272.25 m^2$

$\therefore \quad x^2 = 272.25 m^2$

Taking square root on both sides;

$\sqrt{x^2} = \sqrt{272.25 m^2} = \sqrt{16.5 \times 16.5}$

$\Rightarrow x = 16.5 m$

Now perimeter of square $= 4x$

$\therefore$ Perimeter $= 4 \times 16.5 = 66 m$.

12. Perimeter of a square is 54m. Find its area.

Soln:- Given that perimeter $= 54m$.

Let length of the side $= x$ m

Then perimeter $= 4x$

$\therefore 4x = 54 \Rightarrow x = \frac{54}{4} = 13.5m$

Now area of a square $= (side)^2$

$\therefore$ Area of square $= (13.5m)^2 = 182.25 m^2$

Therefore, the required area $= 182.25 m^2$

EXERCISE 9.4

1. Find the cost of torfing a square shaped garden having one side 19.5m at the rate of Rs. 4 per square meter.

Soln:- Length of the side of square garden = 19.5m

$\because$ Area of square = side x side

$\therefore$ Area of square garden = $19.5 \times 19.5 = 380.25 m^2$

Now rate of torfing per square meter = Rs. 4

$\therefore$ Total cost = 4×380.25

= Rs. 1521

2. Find the cost of turfing a rectangular plot 120m long and 90m wide at the rate of Rs 5.50 per sq m.

Soln:- Length of the plot = 120m

Width of plot = 90m

Area of the plot = length x width —(i)

Putting the values of l and w in Eq(i)

Area of the plot = $120 \times 90 = 10800 m^2$

Cost per square meter = Rs. 5.50

$\therefore$ Total cost of torfing the plot = 5.50×10800

= Rs. 59400

3. A rectangular plot is 46m long and width is 34.5m What will the cost of bricking the plot if 23cm x 11.5cm sized bricks cost Rs. 3500 per 1000 bricks.

Soln:- Length of plot = 46m = 46×100 = 4600 cm

width of plot = 34.5m = 34.5×100 = 3450 cm

Area of plot = $l \times w = 4600 \times 3450$

= $15870000 cm^2$

(246)

- Area of brick = L x W = 23 x 11.5 = 264.5 cm²

Number of bricks = $\frac{\text{Area of plot}}{\text{Area of one brick}} = \frac{15870000}{264.5}$

= 60,000

Cost of per Thousand bricks = 3500

∴ Total Cost = 3500 x 60 ∵ $\frac{60,000}{1000}$

= 210000

∴ Total cost = 21000 Rupees

4. A park is 250 m long. There are 12 flower beds in the park each measuring 2.5m x 1.5m. Find the cost of turfing the remaining area of the park at Rs 0.50 per m² if it is 200 m wide.

Soln: Length of the park = 250 m

Width of the park = 200 m

Total area of the park = 250 x 200 = 50000 m²

Area of one bed = 2.5 x 1.5 = 3.75 m²

Number of beds = 12

Total area of beds = 12 x 3.75 = 45 m²

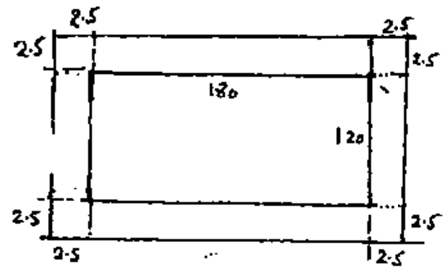
Remaining area of park = 50000 − 45 = 49955 m²

Cost of turfing per m² = 0.50

Total cost = 49955 x 0.50 = Rs. 24977.5

5. A park is rectangular in shape, and is 180 m long and 120 m wide. Find the number of tiles 25 cm x 25 cm required to pave the footpath 2.5 m wide carried around the outside park.

Soln:-



Area of rectangle = 120×180

$= 21600 m^2$

Adding the width of footpath with the rectangle, the new lengths and width will be:

Length = $180 + 2.5 + 2.5 = 185m$

Width = $120 + 2.5 + 2.5 = 125m$

Area of External rectangle = $185 \times 125 = 23125m^2$

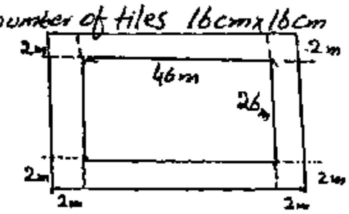
Area of footpath = $23125m^2 - 21600m^2 = 1525m^2$

Area of one tile = $\frac{25}{100} \times \frac{25}{100} = 0.0625m^2$

Number of tiles required = $\frac{\text{Total area of foot path}}{\text{area of one tile}}$

$= \frac{1525}{0.0625} = 24400$

6. A varandah 2m wide was constructed around a hall 46m long and 26m wide. Find the number of tiles 16cm×16cm required for tiling the varandah.



Soln: Area of hall = $26 \times 46 = 1196m^2$

length of external hall = $46 + 2 + 2 = 50m$

width of external hall = $26 + 4 = 30$

Area of external hall = $30 \times 50 = 1500m^2$

Area of Varandah = $1500 - 1196 = 304m^2$

Area of one tile = $\frac{16}{100} \times \frac{16}{100} = 0.0256m^2$

Number of tiles required = $\frac{\text{Total area of varandah}}{\text{area of one tile}}$

$= \frac{304}{0.0256} = 11875$

$\therefore$ Number of tiles required = 11875

(248)

EXERCISE 9.5

1. Find the area of the circle for the following assumptions:

(i) $r = 14\,cm$

Soln:: Here, $r = 14cm$

Area of circle $= \pi r^2$ —(i)

Putting the values in Eq(i)

Area of circle $= \frac{22}{7}(14)^2$

$= \frac{22}{7} \times 196$

$= 22 \times 28$

$= 616\,cm^2$

(ii) $r = 21\,m$

Soln: $r = 21\,m$.

$\because$ Area of circle $= \pi r^2$

$\therefore$ Area of circle $= \frac{22}{7}(21)^2$

$= \frac{22}{7} \times 21 \times 21$

$= 66 \times 21 = 1386 m^2$

(iii) $r = 3.5\,cm$

$\because$ Area of circle $= \pi r^2$

Putting values of π and r

$\therefore$ Area $= \frac{22}{7}(3.5)^2$

$= \frac{22}{7} \times 3.5 \times 3.5$

$= 22 \times 0.5 \times 3.5$

$= 38.5\,cm^2$

(iv) $r = 17.5\,m$

$\because A = \pi r^2$

$\therefore A = \frac{22}{7}(17.5)^2$

$= \frac{22}{7} \times 17.5 \times 17.5$

$= 22 \times 2.5 \times 17.5$

$= 962.5\,m^2$

$\therefore$ The required area is:

$962.5\,m^2$

2. Find the area of semi-circle for the following assumptions:

(i) $r = 9.1\,Cm$

Soln:: As area of circle $= A = \pi r^2$

$\therefore$ Area of circle $= \frac{22}{7} \times (9.1)^2$

$= \frac{22}{7} \times 9.1 \times 9.1$

$= 22 \times 1.3 \times 9.1 = 260.26\,cm^2$

Area of Semi Circle $= \frac{260.26}{2} = 180.13\,cm^2$

(249)

(ii) $r = 16.8m$

Soln: ∵ Area of circle $= A = \pi r^2$

∴ Area of circle $= A = \frac{22}{7}(16.8)^2$

$= \frac{22}{7} \times 16.8 \times 16.8$

$= 22 \times 2.4 \times 16.8 = 887.04\ m^2$

∴ Area of semi-circle $= \frac{887.04}{2} = 443.52\ m^2$

(iii) radius $= 8.4cm$

Soln: As area of a circle $= \pi r^2$

∴ Area of circle $= \frac{22}{7}(8.4)^2$

$= \frac{22}{7} \times 8.4 \times 8.4 = 22 \times 1.2 \times 8.4$

$= 221.88\ cm^2$

So area of semi-circle $= \frac{221.88}{2} = 110.94\ cm^2$

(iv) Radius $= 17.5m$

Since area of a circle $= \pi r^2$

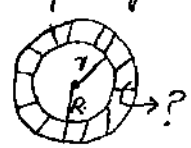
∴ Area $= \frac{22}{7}(17.5)^2 = \frac{22}{7} \times 17.5 \times 17.5$

$= 22 \times 2.5 \times 17.5 = 962.5\ m^2$

⇒ Area of semi-circle $= \frac{962.5}{2} = 481.25\ m^2$

3. Find the area enclosed by concentric for the following:

(i) 8cm, 6cm



Soln: Radius of external circle $= 8\ cm = R$

Radius of internal circle $= r = 6cm$

Enclosed area of concentric circles $= \pi(R^2 - r^2)$

⇒ Area of b/w concentric circles $= \frac{22}{7}\left[(8)^2 - (6)^2\right]$

$= \frac{22}{7}(64 - 36) = \frac{22}{7}(28)$

$= 22 \times 4 = 88\ m^2$

(250)

(ii) 1.6cm, 1.2cm

Soln:- Here; radius of external circle $= R = 1.6cm$

radius of internal circle $= r = 1.2cm$

Area b/w two concentric circles $= \pi(R^2 - r^2)$

$= \frac{22}{7}\left[(1.6)^2 - (1.2)^2\right]$

$= \frac{22}{7}(2.56 - 1.44) = \frac{22}{7}(1.12)$ (0.16)

$= 22 \times 0.16 = 3.52 \ cm^2$

(iii) 9m, 2m

Soln:- Here $R = 9m$, $r = 2m$.

Area b/w concentric circles $= \pi(R^2 - r^2)$

$= \frac{22}{7}\left[(9)^2 - (2)^2\right] = \frac{22}{7}(81 - 4)$

$= \frac{22}{7}(77) = 22 \times 11 = 242 \ m^2$

(iv) 1m, 0.3m

Soln: Here $R = 1m$, $r = 0.3m$

Area b/w concentric circles $= \pi(R^2 - r^2)$

$= \frac{22}{7}\left((1)^2 - (0.3)^2\right)$

$= \frac{22}{7}(1 - 0.09) = \frac{22}{7}(0.91)$ (0.13)

$= 22 \times 0.13 = 2.86 \ m^2$

4. Area of a circle is $616 m^2$. Find its radius.

Sol: Area of a circle $= 616 m^2$

$r = ?$

$\because A = \pi r^2$

$\therefore 616 = \frac{22}{7} r^2$

$\Rightarrow \frac{22}{7} \times \frac{7}{22} \times r^2 = 616 \times \frac{7}{22}$ (28)

$r^2 = 28 \times 7 = 196$

$\Rightarrow \sqrt{r^2} = \sqrt{196} \Rightarrow r = 14 m$

5. Area of a Circle is $78.5m^2$. Find its diameter. ($\pi = 3.14$)

Soln: Area of circle $= 78.5m^2$, $d = ?$

As $A = \pi r^2$

$\therefore 78.5 = 3.14(r^2) \Rightarrow \frac{3.14r^2}{3.14} = \frac{78.5}{3.14}$ (25)

$\Rightarrow r^2 = 25$

Taking square roots on both sides we get

$\sqrt{r^2} = \sqrt{25} \Rightarrow r = 5m$

But $d = 2r$ so, $d = 2 \times 5 = 10m$

6. Area of a semi circle is $56.62cm^2$. Find its radius.

Soln: Area of semi circle $= \frac{\pi r^2}{2}$ — (i)

Putting the values of area and π in eq (i) we get

$56.62 = \frac{3.14r^2}{2}$

$\Rightarrow 3.14r^2 = 2 \times 56.62$ ∵ multiplying b.s by 2

$\Rightarrow \frac{3.14r^2}{3.14} = \frac{2 \times 56.62}{3.14} = 36$

$\Rightarrow r^2 = 36$

$\sqrt{r^2} = \sqrt{36}$ by Taking the Square root

$\Rightarrow r = 6$

$\therefore r = 6cm$

7. Area enclosed by 2 Concentric circles is $2200m^2$ of the radius of outer circle is 40m, Find the radius of inner circle.

Soln: Area b/w two concentric circle $= 2200m^2$

$R = 40m$

$r = ?$

As area b/w concentric circle $= \pi(R^2 - r^2)$

(252)

$\therefore\ 2200 = \pi(R^2 - r^2)$

$\Rightarrow R^2 - r^2 = \frac{2200}{\pi} = \frac{2200}{\frac{22}{7}} = 2200 \times \frac{7}{22} = 700$

$\Rightarrow (40)^2 - r^2 = 700$

$\Rightarrow 1600 - r^2 = 700 \Rightarrow -r^2 = 700 - 1600$

$\Rightarrow -r^2 = -900$ or $r^2 = 900$

Taking square root on both sides

$\sqrt{r^2} = \sqrt{900} \Rightarrow r = 30m$

8. Area enclosed by two concentric circles is $264m^2$. If the radius of inner circle is 4m, find the radius of outer.

Sol: Area b/w concentric circles $= \pi(R^2 - r^2) = 264$ —(i)

$r = 4m$

$R = ?$

Putting $r = 4m$ in (1) we get.

$\pi(R^2 - 4^2) = 264$

$\Rightarrow R^2 - 16 = \frac{264}{\pi}$ ÷ing b.s by 4.

$R^2 - 16 = \frac{264}{22/7} = 264 \times \frac{7}{22}$

$R^2 - 16 = 12 \times 7 = 84$

$\Rightarrow R^2 = 16 + 84 = 100$

Taking the square root on both sides we get

$\sqrt{R^2} = \sqrt{100} \Rightarrow \boxed{R = 10\ m}$

EXERCISE 9.6

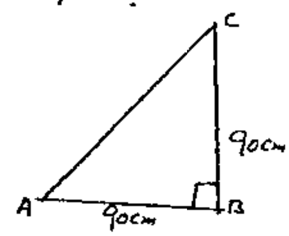
1. A scarf is of the shape of an isoscles right angle triangle, each of equal side is 90 cm. How much cloth will be needed to prepare 8 such scarf? Also find the cost of the cloth at the rate of Rs. 40 per m^2.

Soln: Area of $\triangle ABC = \frac{1}{2} \times \text{Base} \times \text{Altitude}$

$= \frac{1}{2} \times 90 \times 90$

$= 45 \times 90$

$= 4050\ cm^2$



$\therefore$ Area of one scarf $= 4050\ cm^2$

Area of 8 scarf $= 8 \times 4050 = 32400\ cm^2$

Area in meters $= \frac{32400}{100 \times 100} = 3.24\ m^2$

Price of $1m^2$ cloth $= 40$ Rs.

$\therefore$ Total cost $= 3.24 \times 40 =$ Rs: 129.6

2. Sides of a triangular field are in the ratio 5:6:9 If the perimeter of triangle is 240m. Find the area.

Soln: Since the ratio b/w the sides are: 5:6:9

$\therefore$ The sides are $= 5x, 6x, 9x$.

But $a+b+c = 240 \Rightarrow 5x+6x+9x = 240$

$\Rightarrow 20x = 240 \Rightarrow x = \frac{240}{20} = 12m$

$\therefore$ Sides of triangle are: $5x = 5(12) = 60m$

$b = 6x = 6(12) = 72\ m$

$c = 9x = 9(12) = 108\ m$

$\therefore S = \frac{a+b+c}{2} = \frac{24+72+108}{2} = 120m$

According to Hero's formula;

Area of triangle $= \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{120(120-60)(120-72)(120-108)}$

$= \sqrt{120 \times 60 \times 48 \times 72}$

$= \sqrt{60 \times 60 \times 2 \times 12 \times 12 \times 2 \times 2}$

$= 60 \times 12 \times 2\sqrt{2} = 1440\sqrt{2}\ m^2.$

So The required area $= 1440\sqrt{2}\ m^2$

3. The front curtain of a tent is triangular in shape. How much Rexene will be required to prepare the curtain having sides 5m, 5m and 8m

Soln: Let the sides of the triangle be: a, b and c

Then $a = 5m$, $b = 5m$, $c = 8m$

$\because S = \frac{a+b+c}{2}$ —— (i)

Putting the values of a, b and c in Eq (i) we have

$S = \frac{5+5+8}{2} = \frac{18}{2} = 9$

According to Hero's formula.

Area of triangle $= \sqrt{S(S-a)(S-b)(S-b)}$ —— (ii)

Putting the values of S and a, b, c in Eq (ii)

Area $= \sqrt{9(9-5)(9-5)(9-8)}$

$= \sqrt{9 \times 4 \times 4 \times 1} = 3 \times 4 = 12m^2$

So the required area $= 12m^2$.

Q4. Diagonal of a quadrilateral plot is 65m. Its distance from the opposite vertices is 44m and 52m. Find the area of the plot. Also find the cost of its levelling at Rs. 2.50 per m^2.

(255)

Soln: In the given fig;

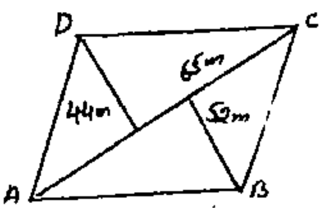
Area of $\triangle ABC = \frac{1}{2} \times$ Base $\times$ Altitude

$= \frac{1}{2} \times 65 \times 52 = 65 \times 26 = 1690\, m^2$

Area of $\triangle ACD = \frac{1}{2} \times 65 \times 44$

$= 65 \times 22$

$= 1430\, m^2$



Area of quadrilateral ABCD = Area of $\triangle ABC$ + Area of ACD

$= 1690 + 1430 = 3120\, m^2$

Cost of levelling the ground per m^2 = 2.50 Rs.

Total cost = $2.50 \times 3120 = 7800$

= Rs: 7800

5. A field is in the shape parallelogram. Its base is 750m and hight is 360m. What will be the revenue amount if it is based out at Rs: 9000 per hectare. (1 Hect = 10000 m^2)

Soln: The figure is given. From it

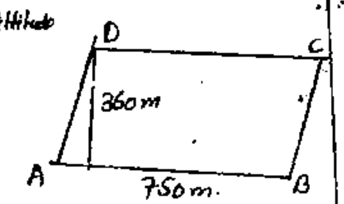
Area of parallelogram ABCD = Base $\times$ Altitude

$= 360 \times 750$

$= 270000\, m^2$

So total area in Hectare $= \frac{270000}{10000}$

= 27 Hectare



Revenue of one Hectare = 9000 Rs

Total revenue = 27×9000 = Rs: 243000

6. The area of courtyard, parallelogram shape is 39 m^2. Find its base if its hight is 5.2 m.

(256)

Soln. In the parallelogram ABCD;

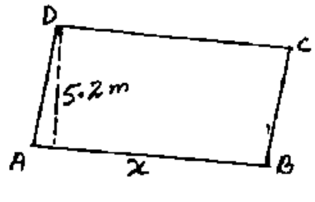
Area of parallelogram ABCD = Base x Altitude ——(i)

But Area = $39 m^2$

Altitutade = $5.2m$

Let base = x m

Putting these values in Eq(i)



$39 = x \times 5.2$

÷ing both sides by 5.2 we get

$\frac{x \times 5.2}{5.2} = \frac{39}{5.2}$

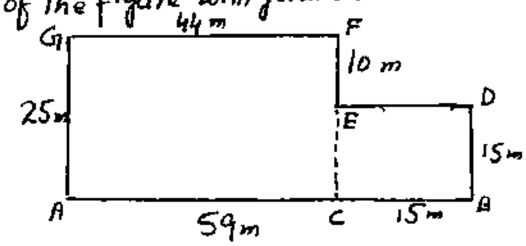
$x = 7.5m$

So the required base = 7.5m.

7. Find the area of the figure with given dimensions.

Sol:

Name the sides of the fig



Area of rectangle ACFG = $44 \times 25 = 1100 m^2$

Area of Square BCDE = $15 \times 15 = 225 m^2$

∴ Total area of given figure = $1100 + 225$

$= 1325 m^2$

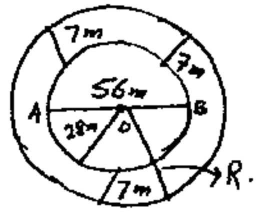
8. Diameter of a circular garden is 56m. A circular pavement of width 7m is constructed around it. Find the expenditure of its construction at Rs. 50 per m^2.

(257)

Sol: According to the fig;

Here, $r = \frac{56}{2} = 28m$

$R = 28 + 7 = 35m$



Area b/w two concentric circles is

$= \pi(R^2 - r^2)$

$= \frac{22}{7}\left((35)^2 - (28)^2\right)$

$= \frac{22}{7}[1225 - 784] = \frac{22}{7}(441)$ 63

$= 22 \times 63 = 1386m^2$

Cost per square meter = Rs. 50

Total cost = 50×1386 = Rs. 69300

9) Diameter of two concentric circles are 160cm and 90cm respectively. Find the area inclosed by them.

Sol: Here $R = \frac{160}{2} = 80cm$

$r = \frac{90}{2} = 45cm$

Area b/w centric circle $= \pi(R^2 - r^2)$

$= \frac{22}{7}\left[(80)^2 - (45)^2\right]$

$= \frac{22}{7}[6400 - 2025]$

$= \frac{22}{7}(4375)$ 625

$= 22 \times 625 = 13750 cm^2$

10. A field, rectangular in shape, is based out at the rate of Rs. 7.50 per m². Find the total revenue, if the field is 350m long and 280m wide.

(258)

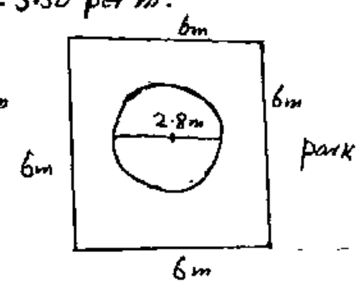
Sol: Length of field = 350m
Width of field = 280m
Total area of field = $L \times W = 350 \times 280 = 98000 m^2$
revenue per m^2 = Rs. 7.50
Total revenue = $7.50 \times 98000 = 735000$
So total revenue = Rs. 735000

11. Arshad covered a distance of 2800m by taking 4 rounds of a square park. Find the area of park.

Sol: Total rounds = 4
Distance covered of 4 rounds = 2,800m
perimeter of square park = $\frac{2800}{4} = 700$
But Perimeter = $4x$
$\therefore 4x = 700 \Rightarrow \frac{4x}{4} = \frac{700}{4}$
$\therefore$ Side of a square = $x = 175m$
$\therefore$ Area of a square park = $\times 175 = 30625 m^2$

12. A circular fountain, having diameter 2.8m, is situated in the middle of a square shapped park. Whose one side is 6m long. Find the area of the fountain. Also find the cost of turfing the park at Rs. 5.50 per m^2.

Sol: Diameter of fountain = 2.8m
Radius of fountain = $\frac{2.8}{2}$
$= 1.4m$



Area of fountain = πr^2
$= 3.14(1.4)^2 = 3.14 \times 1.96 = 6.16 m^2$

Area of park = $6 \times 6 = 36 m^2$

Area for turfing the park = $36 - 6.16 = 29.8 m^2$

Cost per m^2 = 5.50 Rs.

Total cost = $29.8 \times 5.50 = 164.12$

$\therefore$ Total cost for turfing the park = Rs: 164.12.

13. A rectangular park is divided into square shaped flower beds having side of 3m. How many flower beds can be made if the park is 21m long and 18m wide.

Sol: Length of the park = 21m

Width of the park = 18m

Area of rectangular park = $21 \times 18 = 378 m^2$

One side of a square bed = 3m

Area of one square bed = $3 \times 3 = 9 m^2$

Number of square beds = $\frac{\text{Total area of park}}{\text{Area of one bed}}$

$= \frac{378}{9} = 42$

$\therefore$ Number of square beds = $42 m^2$

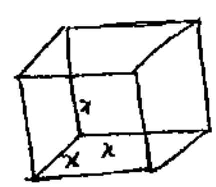
Cube:- The edges of a cube are equal in length.

Volume of cube = edge × edge × edge

$= x \cdot x \cdot x$

$= x^3 m^3$

$= (Side)^3$



Volume of cuboid

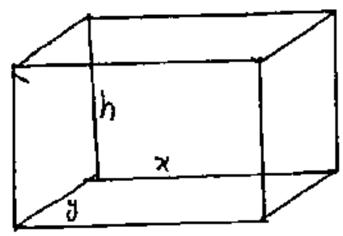
Let length of cuboid is x, width is y and hight is h
then volume of cuboid $= L \times W \times h$

$= xy \times h \qquad \because xy = \text{Area}$

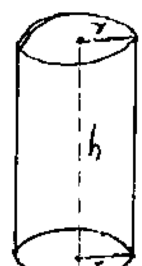
$= \text{Area} \times h$

$= Ah$

Cuboid



Volum of right circular cylinder:



Volume of right circular cylinder $= \pi r^2 h$
where 'h' is hight and 'r' is radius of the base.

Volume of right circular Cone:

Volume of right circular cone $= \frac{1}{3}\pi r^2 h$

Volume of Sphere:

Sphere



Volume of sphere $= \frac{4}{3}\pi r^3$

and

Volume of Hemisphere $= \dfrac{4/3\,\pi r^3}{2}$

$= \frac{2}{3}\pi r^3$



EXERCISE 9.7

1. Find the volume of the cubes having the following edges:

(i) 6mm

Soln: Edge of cube = 6mm,

$$\text{Volume of a cube} = (\text{edge})^3$$

$$= (6\,mm)^3 = 216(mm)^3$$

(ii) edge = 8.5cm

$\because$ Volume of a cube = $(\text{edge})^3$

$$= (8.5)^3 = 614.25\,cm^3$$

2. Find the volume of cuboids having the following edges:

(i) Length = 6dm, Width = 5dm, Hight = 4dm

As, volume of cuboids = Length × width × hight ——(i)

Putting the values of l, w, h in Eq(i) we get.

$$\text{volume of cuboids} = 6 \times 5 \times 4 = 120\,dm^3$$

(ii) $l = 3.5cm$, $w = 2.4cm$, $h = 2cm$.

Sol: Here, $l = 3.5cm$, $w = 2.4cm$, $h = 2cm$

Volume of cuboid = $l \times w \times h$

$\therefore$ Volume of cuboid $= 3.5 \times 2.4 \times 2 = 16.8\,cm^3$

3. Find the measure of the edge of cube, when its volume is:

(i) $343\ cm^3$

Soln: Here volume $= 343\,cm^3$ ——(i)

Let one edge $= x$

$\therefore$ Volume $= x^3$ ——(ii)

Comparing Eq (i) and (ii) we get

$$x^3 = 343 = 7 \times 7 \times 7 = 7^3 \Rightarrow \boxed{x = 3\,cm} \text{ Ans}$$

$x^3 = 7^3$
$(x^3)^{1/3} = (7^3)^{1/3}$
$x = 7$

(261)

(ii) $1.728\ dm^3$

Soln: Let edge of the cube $= x$

Volume of cube $= x^3$ ——(i)

But volume of cube $= 1.728 dm^3$ ——(ii)

Comparing (i) and (ii) we get.

$x^3 = 1.728 \Rightarrow x^3 = 1.2 \times 1.2 \times 1.2$

$\Rightarrow x^3 = (1.2)^3 \Rightarrow x = 1.2$

$\therefore$ Edge of cube $= 1.2 dm$

4. A box is 0.4m long, 0.5m wide and 0.4m high. Find its volume.

Sol: Given That,

Length of box $= 0.9m$

Width of box $= 0.5m$

hight of box $= 0.4m$

$\because$ Volum of box $= L \times W \times h$

$\therefore$ Volume of box $= 0.9 \times 0.5 \times 0.4 = 0.18 dm^3$

5. Volume of cuboid is $140\ m^3$. Find its hight if its base is $35m^2$

Sol: Volume of cuboid $= 140\ m^3$

Area of cuboid $= 35m^2$

hight of cuboid $= h = ?$

As, volume of cuboid $=$ Base area $\times h$

$\Rightarrow 140 = 35 \times h$

$\Rightarrow \frac{35 \times h}{35} = \frac{140}{35}{}^{7} \Rightarrow \boxed{h = 7\ m}$

6. Find the volume of cylindrical drum having radius 0.5m and hight 1.75m

Sol: Here $r = 0.5m$, $h = 1.75m$, Volume $= ?$

Volume of cylindrical drum = $\pi r^2 h$ —(i)

Putting the values of π, r and h in Eq (i)

∴ Volume $= \frac{22}{7}(0.5)^2 \times 1.75$

$= \frac{22}{7} \times 0.25 \times 175$

$= 22 \times 0.25 \times 25 = 137.5\ m^3$

7- Find the width of cuboid if its hight is 5m, length 8m and volume $260 m^3$.

Sol: Width of cuboid = ? = x

hight of cuboid = 5m

Length of cuboid = 8m

Volume of cubiod = $260 m^3$

∵ Volume of cuboid = $l \times w \times h$

⇒ $260 = 8 \times x \times 5$

⇒ $260 = 40 \cdot x \Rightarrow \frac{260}{40} = \frac{40 \cdot x}{40}$

⇒ $x = \frac{260}{40} = 6.5m$

So width of cubiod = 6.5m.

8- Find the hight of the cylinder, if its volume is $6050 m^3$ and radius of its base is 5m.

Sol: Volume of cylinder = $6050\ m^3$

Radius of base = 5m

hight of cylinder = h = ?

As volume of cylinder = $\pi r^2 h$

⇒ $6050 = \frac{22}{7} \times (5)^2 \times h$

$6050 \times 7 = \frac{22}{7} \times 7 \times 25 \times h$

$\frac{42350}{550} = \frac{550 \times h}{550}$ ÷ing b.s by 550

⇒ $h = 77 m$.

9. The volume of right circular cone is $154 m^3$. Find the radius of its base if its hight is 12m.

Sol: Volume of right circular cone $= 154 m^3$

Let required base $= x$

hight of cone $= 12 m$

we know that,

Volume of right circular cone $= \frac{1}{3}\pi r^2 h$

$\therefore \quad 154 = \frac{1}{3} \times \frac{22}{7} (r)^2 \times 12$

$\frac{154}{88} = \frac{88}{88} \times r^2$

$\Rightarrow r^2 = \frac{154}{88} = 12.25$

Taking the squareroot on both sides we get

$\sqrt{r^2} = \sqrt{12.25} \Rightarrow r = 3.5 m.$

10. The volume of right circular cone is $4312 dm^3$. If ratio b/w radius and hight is 2:3, find radius and hight.

Sol: Volume of cone $= 4312$

radius : hight $= 2:3 \Rightarrow \frac{r}{h} = \frac{2}{3}$

$\Rightarrow 3r = 2h$ $\quad\because$ by Cross multiplication

$\Rightarrow r = \frac{2h}{3}$ —(i)

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{2h}{3}\right)^2 \times h$

$\Rightarrow \quad 4312 = \frac{22}{7} \times \frac{4h^2}{9 \times 3} \times h$

$4312 = \frac{22}{7} \times \frac{4}{27} \times h^3$

$\Rightarrow h^3 = \frac{4312 \times 7 \times 27}{88} = 9261$

$\Rightarrow h^3 = 21 \times 21 \times 21 = (21)^3$

$\Rightarrow \boxed{h = 21 dm}$

Eq (i) $\Rightarrow \quad r = \frac{2 \times 21}{3} = 2 \times 7 \Rightarrow \boxed{r = 14 dm}$

(265)

11. Find the volume of hemisphere when its radius is:

(i) $5\frac{1}{4}$ dm

Sol: $r = 5\frac{1}{4}\ dm = \frac{21}{4}\ dm$

Volume of Hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{21}{4}\right)^3$

$= \frac{44}{21} \times \frac{21 \times 21 \times 21}{4 \times 4 \times 4}$

$= \frac{44 \times 21 \times 21}{64} = \frac{4851}{16} = 303\frac{3}{16}\ dm$

∴ Volume of hemisphere $= 303\frac{3}{16}$ dm

(ii) $r = 10\frac{1}{2}\ cm = \frac{21}{2}\ cm.$

Sol: Volume of hemisphere $= \frac{2}{3}\pi r^3$ —— (i)

Putting the values of π and r in Eq (i).

Volume of hemisphere $= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^3$

$= \frac{44}{21} \times \frac{21 \times 21 \times 21}{8_2}$

$= \frac{11 \times 21 \times 21}{2} = \frac{4851}{2} = 2425\frac{1}{2}\ cm^3$

12. Find the volume of a sphere to the nearest hundredths when its radius is: (Take $\pi = 3.14$)

(i) $r = 0.42\ m$

Soln: $r = 0.42m$

Volume of Sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times 3.14 \times (.42)^3$

$= \frac{4 \times 3.14 \times 0.42 \times 0.42 \times 0.42}{3}$

$= 0.31\ m^3$

(ii) $r = 0.6cm$

Sol: Volum of Sph $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times 3.14 \times (0.6)^3$

$= \frac{4}{3} \times 3.14 \times 0.6 \times 0.6 \times 0.6$

$= \frac{4 \times 3.14 \times 0.216}{3}$

$= 0.9cm^3.$

EXERCISE : 9.8

1. A sewerage fit, having the shape of a cube, was dug with a side measuring 12m. In how many turns, three trucks will be able to take away the soil if $19.2\ m^3$ soil can be loaded on one truck.

Sol: Depth of sewrage = 12m.

Volume of cube $= (12m)^3 = 12 \times 12 \times 12 = 1728\ m^3$

Volume of one truck soil $= 19.2 m^3$

Volum of soil loaded in 3 trucks $= 3 \times 19.2 = 57.6 m^3$

Number of turns $= \frac{1728}{57.6} = 30$

So turns for 3 trucks = 30

2. A solid cube is made of a metal. What will be the volume of a cube if one of its sides $4\frac{1}{2}$ cm. Find its weight if $1 cm^3$ of the metals is 16 grams.

Sol: Edge of solid cube $= 4\frac{1}{2} cm = \frac{9}{2} = 4.5 cm$

Volume of metal cube $= (4.5 cm)^3 = 91.125 cm^3$

$\because$ Weight of $1 cm^3$ = 16 grams

$\therefore$ Total weight of cube $= 16 \times 91.125 = 1458 g$

Therefore weight of metal cube = 1458 g

3. Tariq bought a piece of wood 2m long, 35cm wide and 25cm thick. What will it cost him if the rate of wood Rs. 24000 per m^3?

Sol: Length of piece of wood = 2 m

Width of piece of wood $= 35 cm = \frac{35}{100} = 0.35 m$

thickness of piece of wood $= 25 cm = \frac{25}{100} = 0.25 m$

Volume of piece $= l \times w \times h = 2 \times 0.35 \times 0.25$

$= 0.175 m^3$

(267)

∵ Price of wood piece per m^3 = Rs.24000

∴ Total price of piece = 24000×0.175 = Rs. 4200

4. Find the total cost on constructing a stage for a function at Rs. 1500 per m^3, if the stage is 8m long, 5m wide and 1m high.

Sol: Given that $l = 8\,m$, $w = 5m$ and $h = 1m$.

Volume of stage = $l \times w \times h = 8 \times 5 \times 1 = 40\,m^3$

∵ Cost per m^3 = 1500

∴ Total cost = 1500×40 = Rs. 60,000.

5. Two cylindrical drums of kerosene oil have the same hight of 1·5m, and are made of same meterial. The diameters are 1·2m and 1·6m. Find the ratio of their volume.

Sol: Hight of 1st cylinder = $h_1 = 1.5m$

diameter of 1st cylinder = $d_1 = 1.2m$

radius of 1st cylinder = $\frac{d_1}{2} = \frac{1.2}{2} = 0.6m$

Volume of 1st cylinder = $V_1 = \pi r^2 h = \pi (0.6)^2 \cdot 1.5$

$= 0.54\pi\, m^3$

Similerly ;

Hight of 2nd cylinder = $h_2 = 1.5m$

$d_2 = 1.6m$

$r_2 = \frac{1.6}{2} = 0.8\,m$

Volume of 2nd Cylinder $= \pi(.8)^2(1.5)$

$= 0.96\pi\, m^3$

∴ $V_1 : V_2 = 0.54\pi m^3 : 0.96\pi m^3$

$= 0.54 : 0.96$ ÷ing by π

$= 54 : 96$ xing by 100

$= 9 : 16$ ÷ing by 6

So ratio b/w volume is $9 : 16$

(268)

6. What will be the cost of digging 21m deep well having a diameter of 2m at Rs. 300 per m^3.

Sol: depth of well = 21m

Diameter of well = 2m $\Rightarrow r = \frac{2}{2} = 1m$

Volume of the well $= \pi r^2 h$

$= \frac{22}{7}(1)^2 \cdot 21 = 22 \times 3 = 66 m^3$

$\because$ Cost of per m^3 for digging = Rs300

$\therefore$ Total cost of digging a well $= 66 \times 300 =$ Rs. 19800

7. A cone shaped minarates is 12dm high and has a diameter of 14dm. Find its volume.

Sol: hight of cone shaped minarates = 12dm

radius of minarates $= \frac{14}{2} = 7dm$

Volume of minarates $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 12$

$= \frac{22 \times 7 \times 12}{3} = 88 \times 7 = 616 dm^3$

$\therefore$ Volume of minarates $= 616 dm^3$.

8. Find the volume of decorative cap, having a shape of cone, with a radius of 2.1cm and hight 10cm.

Sol: Radius = 2.1cm, h = 10cm

Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (2.1)^2 \times 10$

$= \frac{22 \times 2.1 \times 2.1 \times 10}{3 \times 7} = 22 \times 0.21 \times 10$

$= 46.2 cm^3$

$\therefore$ The required volume $= 46.2 cm^3$

9. Find the volume of children's football with radius of 10.5cm.

Sol: Radius of football ground = 10.5 cm

Volume of ground = $\frac{4}{3}\ 4(\pi r^3)$

$= \frac{4}{3} \times \frac{22}{7} \times (10.5)^3$

$= \frac{88}{21} \times 1157.63$

$= 4851 cm^3$

∴ The required volume = $4851\ cm^3$

10. Dome of a masjid is hemisphere in shape. Find its volume to the nearest tenths if its diameter 2.1m.

Sol: radius of masjid dome = $\frac{2.1}{2} = 1.05m$

Volume of dome = $\frac{2}{3}\pi r^3$. ∵ Dome is a hemisphere.

$= \frac{2}{3} \times \frac{22}{7} \times (1.05)^3$

$= \frac{44}{21} \times 1.157 = \frac{50.4355}{21} = 2.425$

∴ Volume of the dome of mosque = $2.425\ m^3$

11. A metallic sphere is created in a fountain. Find its volume when its diameter is $5\frac{1}{4}$ dm.

Sol: Radius of metallic sphere = $\frac{21}{4} \div \frac{2}{1} = \frac{21}{4} \times \frac{1}{2} = \frac{21}{8} dm$

Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \left(\frac{21}{8}\right)^3$

$= \frac{88}{21} \times \frac{21 \times 21 \times 21}{8 \times 8 \times 8}$

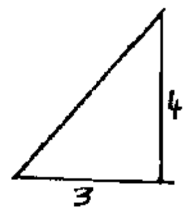
$= \frac{11 \times 21 \times 21}{64} = 75.79\ dm^3$

∴ The required volume = $75.79\ dm^3$

EXERCISE : 9.9

1. Find the area of following shapes.

(i) In the given triangle;



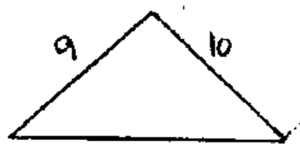
Base = 3, Altitude = 4

Area of triangle = $\frac{1}{2}$ × Base × Altitude

$= \frac{1}{2} \times 3 \times 4$

$= 6\ cm^2$

(ii) In this triangle,



$a = 9,\ b = 10,\ c = 17$

$\because S = \frac{a+b+c}{2}$

$\therefore S = \frac{9+10+17}{2} = \frac{36}{2} = 18$

By Hero's formula

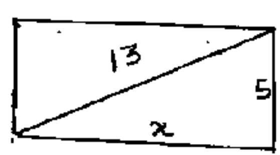
Area of triangle $= \sqrt{S(s-a)(s-b)(s-c)}$

$= \sqrt{18(18-9)(18-10)(18-17)}$

$= \sqrt{18 \times 9 \times 8 \times 1} = \sqrt{2 \times 9 \times 9 \times 2 \times 4}$

$= 2 \times 2 \times 9 = 36\ cm^2$

iii. The given figure is a rectangle. Firstly we will find its length by using Pythagorus Theorem.



$x^2 + 5^2 = 13^2 \Rightarrow x^2 + 25 = 169 \Rightarrow x^2 = 169 - 25 = 144$

$\Rightarrow \sqrt{x^2} = \sqrt{144} \Rightarrow x = 12$

$\therefore$ Area of rectangle $= l \times w = 12 \times 5 = 60 cm^2$

$\therefore$ Area of given fig $= 60\ cm^2$

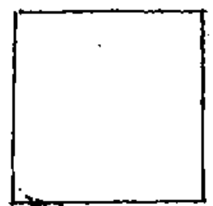
(271)

(iv) The given fig is square of side 5.5cm

∴ Area of square = Side x Side

= 5.5 x 5.5

= 30.25 cm²



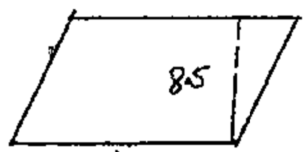
(v) The given fig is a parallelogram.

Here base = 13 cm, h = 8.5

∵ Area ||gram = Base x h

= 13 x 8.5

= 110.5 cm²

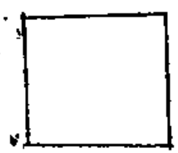


vi) The given fig is a square of side 5cm

∴ Area of square = side x Side

= 5 x 5

= 25 cm²



2. Find the volume of the following figures:

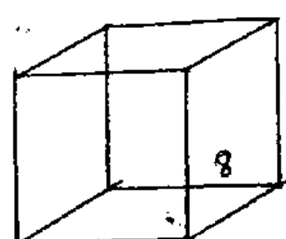
(i) Sol: The given figure is cube.

∴ Volume of cube = (side)³

= (8)³

= 8x8x8

= 512 cm³

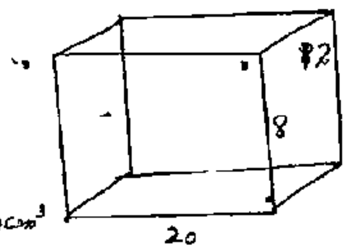


Therefore, The required volume of the given figure = 512cm³.

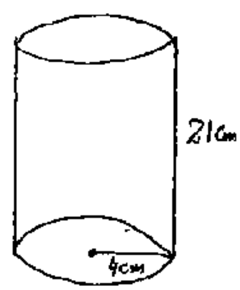
ii) The given fig is cubied.

∵ Volume $= L \times W \times h$
$= 20 \times 12 \times 8$
$= 240 \times 8$
$= 1920\,cm^3$

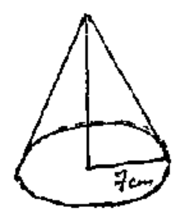
So the required volume $= 1920\,cm^3$



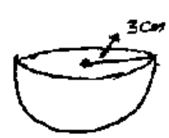
iii) The given figure is right circular Cylinder with $h = 21cm$ and $r = 4cm$.
So its volume is given by;
volume $= \pi r^2 h$
$= \frac{22}{7} (4)^2 \times 21$
$= 22 \times 16 \times 3 = 1056\,cm^3$



iv) The given fig is right circular cone where $r = 7cm$ and $h = 24cm$
∴ volume $= \frac{1}{3}\pi r^2 h$
$= \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 24$
$= \frac{1}{3} \times \frac{22}{7} \times 49 \times 24$
$= 22 \times 7 \times 8 = 1232\,cm^3$



v) The given fig is hemisphere with $r = 3cm$
∴ Volume of hemisphere $= \frac{2}{3}\pi r^3$
$= \frac{2}{3} \times \frac{22}{7} \times (3)^3$
$= \frac{2 \times 22 \times 27}{21} = 56.57\,cm^3$



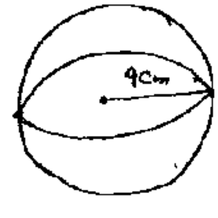
(273)

(vi) The given fig is sphere with $r = 9$ cm. Therefore, its

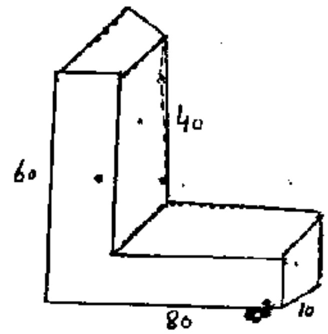
Volume $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(9)^3 = \frac{4}{3}\pi \times 9 \times 9 \times 9$

$= 12 \times 81\pi = 972\pi \text{ cm}^3$



vii: The given fig is divided into two parts and will finds the volume of each parts separately.



$V_1 = l \times w \times h$

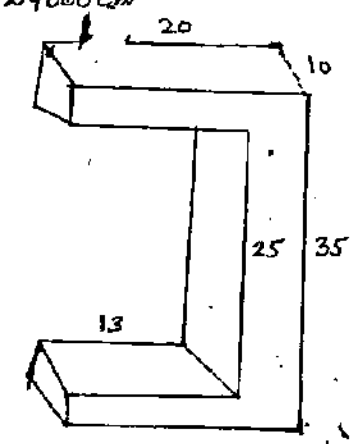
$= 20 \times 10 \times 40$

$= 8000 \text{ cm}^3$

$V_2 = 80 \times 20 \times 10 = 16000$

$V = V_1 + V_2 = 8000 + 16000 = 24000 \text{ cm}^3$

viii) The given figure is divided into three parts.



Volume of upper part:

$V_1 = 5 \times 13 \times 10 = 650 \text{ cm}^3$

Volume of middle part:

$V_2 = 10 \times 7 \times 35$

$= 2450 \text{ cm}^3$

Volume of Lower part:

$V_3 = 5 \times 13 \times 10 = 650 \text{ cm}^3$

$\therefore V = V_1 + V_2 + V_3 = 650 + 2450 + 650 = 3750 \text{ cm}^3$

$\therefore$ Total volume of given fig $= 3750 \text{ cm}^3$

REVIEW EXCERCISE 9

Select the correct answer from the given options.

Ans

1) A	2) B	3) B	4) D
5) C	6) B	7) B	8) C
9) B	10) D	11) A	12) A
13) D	14) B	15) B	16) B
17) A	18) C	19) A	20) B
21) C	22) D	23) A	24) D

End of chap # 9

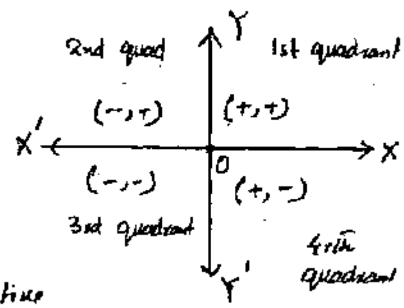
(275)

Unit 10

INTRODUCTION TO COORDINATE GEOMETRY

Carthesian plane:- When two lines XOX' and YOY' intersect each other at right angle at point O, then the plane is divided into four parts. Each part is called quadrant. XOX' is called X-axis, YOY' is called Y-axis and point 'O' is called origin.

The quadrants are called 1st, 2nd, 3rd and 4th quadrants



(i) In 1st quadrant both the coordinates of X and Y are positive.

(ii) In 2nd quadrant, the X coordinate will be negative and Y-coordinate will be positive

(iii) In 3rd quadrant both will be negative.

(iv) In 4rth, X will be positive while Y will be negative.

Furthermore, The Y coordinate on X-axis will be zero and vice versa.

Distance Formula

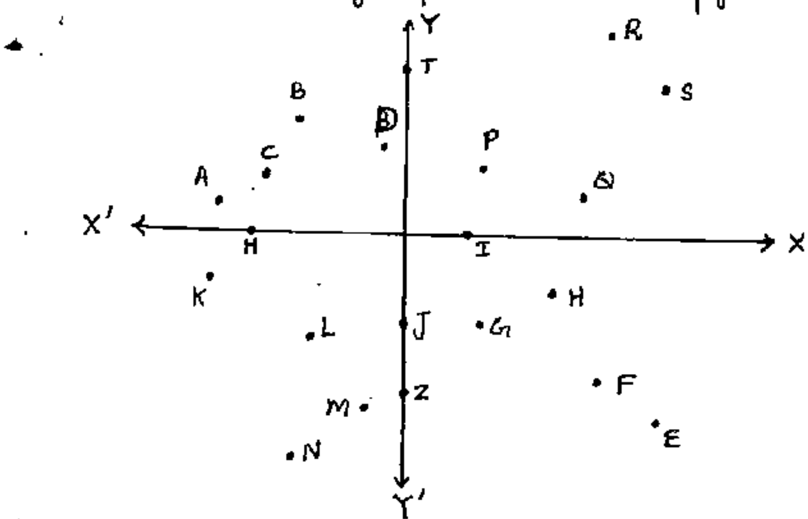
Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points on plane, then the distance between P and Q is

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

(276)

EXERCISE 10.1

1) Find the coordinates of the points shown in the figure.



Sol: The coordinates of the points are:

A$(-10,2)$, B$(-6,6)$, C$(-7,3)$, D$(-1,5)$

E$(9,-9)$, F$(7,-6)$, G$(2,-4)$, H$(5,-2)$

I$(3,0)$, K$(-3,-9)$, L$(-4,-5)$, M$(-2,-8)$

N$(-5,-10)$, P$(4,4)$, Q$(8,2)$, R$(4,10)$, S$(9,8)$, T$(0,9)$.

2. Determine the x and y coordinates of the following points and then plot the points.

Soln: (i) $A(-2,5) \Rightarrow x=-2,\ y=5$

(ii) $B(3,0) \Rightarrow x=3$ and $y=0$

(iii) $C(6,-7) \Rightarrow x=6,\ y=-7$

(iv) $D(-4,-2) \Rightarrow x=-4,\ y=-2$

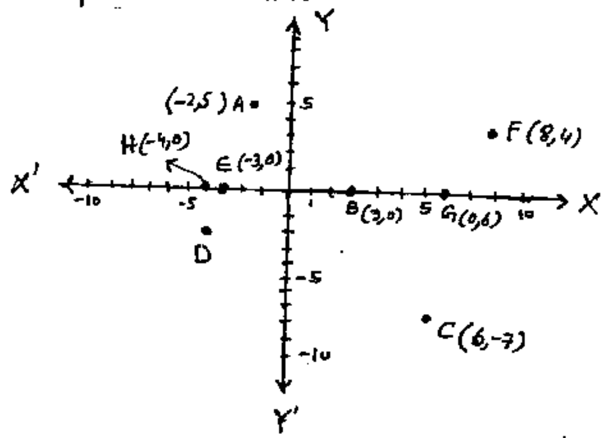
(v) $E(0,-3) \Rightarrow x=0,\ y=-3$

(vi) $F(8,4) \Rightarrow x=8,\ y=4$

(vii) $G(0,6) \Rightarrow x=0,\ y=6$

(viii) $H(-4,0) \Rightarrow x=-4$ and $y=0$

Graphycal representation is below:



3. In which quadrant, the following points are lying?

(i) $(7,\sqrt{5})$ First quadrant (ii) $(5, 9.5)$ 1st quadrant

(iii) $(\sqrt{2}, -4.1)$ 4rth quadrant (iv) $(-2.9, -3.6)$ 3rd quadrant

(v) $(-3/5, -1)$ 3rd quadrant (vi) $(2\frac{3}{4}, -5)$ 4rth quadrant

(vii) $(-6, \sqrt{3})$ 2nd quadrant (viii) $(1.49, -2.5)$ 3rd quadrant

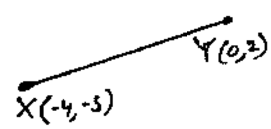
(ix) $(9,0)$ On X-axis (x) $(0,-2.5)$ on Y-axis.

4. Find the distance b/w the following points:

(i) $X(-4,-5)$, $Y(0,2)$

Soln: Distance formula is

$$|XY| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \quad \text{---(i)}$$



Here $x_2 = 0, x_1 = -4, y_2 = 2, y_1 = -5$

Putting these values in Eq (i) we get

$$|XY| = \sqrt{(0-(-4))^2 + (2-(-5))^2} = \sqrt{(0+4)^2 + (2+5)^2}$$

$$= \sqrt{16 + 7^2} = \sqrt{16+49}$$

$$= \sqrt{65}$$

(278)

(ii) $X(3,7)$, $Y(-5,1)$

Sol: Here $x_2 = -5$, $x_1 = 3$, $y_2 = 1$, $y_1 = 7$

According to distance formula:

$$|XY| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-5-3)^2 + (1-7)^2} \quad \because \text{Putting the values}$$

$$= \sqrt{(-8)^2 + (-6)^2}$$

$$= \sqrt{64+36} = \sqrt{100} = 10$$

iii) $X(4,0)$, $Y(0,3)$

Sol: Here $x_2 = 0$, $x_1 = 4$, $y_2 = 3$, $y_1 = 0$

According to distance formula:

$$|XY| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0-4)^2 + (3-0)^2} \quad \because \text{Putting the values}$$

$$= \sqrt{(-4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

iv) $X(-1,9)$, $Y(3,4)$

Sol: Here $x_2 = 3$, $x_1 = -1$, $y_2 = 4$, $y_1 = 9$

According to distance formula:

$$|XY| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)}$$

$$= \sqrt{[3-(-1)]^2 + (4-9)^2} \quad \because \text{By putting the values}$$

$$= \sqrt{(3+1)^2 + (-5)^2}$$

$$= \sqrt{(4)^2 + 25}$$

$$= \sqrt{16+25}$$

$$= \sqrt{41}$$

(279)

(v) $X(-7,2)$, $Y(5,7)$

Sol: Here $x_2=5$, $x_1=-7$, $y_2=7$, $y_1=2$

According to distance formula:

$$|XY| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(5-(-7))^2+(7-2)^2}$$
$$= \sqrt{(5+7)^2+(5)^2} = \sqrt{(12)^2+25}$$
$$= \sqrt{144+25} = \sqrt{169} = 13$$

(vi) $X(-4,12)$, $Y(6,-12)$

Sol: Here, $x_2=6$, $x_1=-4$, $y_2=-12$, $y_1=12$

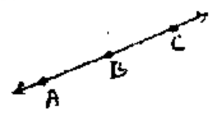
Distance formula is:

$$|XY| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(6-(-4))^2+(-12-12)^2}$$
$$= \sqrt{(6+4)^2+(-24)^2}$$
$$= \sqrt{(10)^2+576}$$
$$= \sqrt{100+576} = \sqrt{676}$$
$$= \sqrt{2\times2\times13\times13}$$
$$= \sqrt{2^2\times13^2}$$
$$= \sqrt{2^2}\times\sqrt{13^2}$$
$$= 2\times13 = 26$$

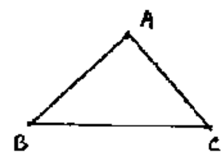
2	676
2	338
13	169
13	13
	1

<u>Collinear points</u>: Three or more points are said to be collinear if they lies on same line. e.g in the given fig, A, B, C are collinear

Remarks: (i) If A, B, C are collinear then
$|AB| + |BC| = |AC|$

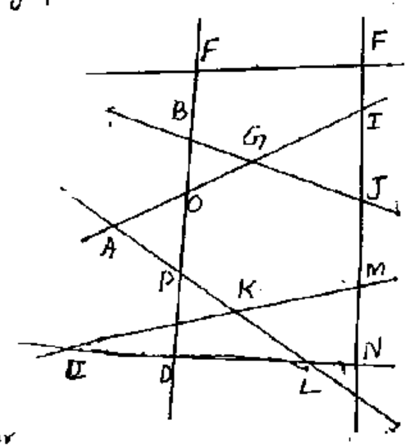


(ii) If A, B, C are not collinear then $\triangle ABC$ will formed and the sum of two sides of the triangle is always greater than the 3rd side i.e
$|AB| + |AC| > |BC|$



EXERCISE 10.2

1. Use the figure to indicate the collinear and non-collinear from the following points



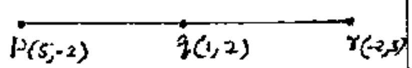
(i) (O, G, I) Collinear
(ii) (F, F, G) Non collinear
(iii) (M, N, O) Non collinear
(iv) (F, B, D) Non collinear
(v) (E, H, I) Non collinear
(vi) (J, G, B) Collinear
(vii) (A, O, P) Non collinear
(viii) (C, D, L) Collinear
(ix) (H, I, J) Non collinear
(x) (B, C, D) Non collinear

(281)

2. Show that the points $P(5,-2)$, $q(1,2)$, $r(-2,5)$ are collinear.

Sol:

$|Pq| = \sqrt{(1-5)^2 + (2-(-2))^2}$



$= \sqrt{(-4)^2 + (2+2)^2}$

$= \sqrt{16+16} = \sqrt{32}$

$= \sqrt{16\times 2} = 4\sqrt{2}$

$|qr| = \sqrt{(-2-1)^2 + (5-2)^2} = \sqrt{(-3)^2 + (3)^2}$

$= \sqrt{9+9} = \sqrt{18} = \sqrt{9\times 2}$

$= 3\sqrt{2}$

$|Pr| = \sqrt{(-2-5)^2 + (5-(-2))^2}$

$= \sqrt{(-7)^2 + (7)^2} = \sqrt{49+49} = \sqrt{98} = \sqrt{49\times 2}$

$= 7\sqrt{2}$

Now; $|Pq| + |qr| = 4\sqrt{2} + 3\sqrt{2} = (4+3)\sqrt{2} = 7\sqrt{2} = |Pr|$

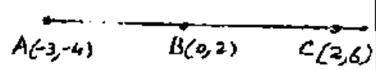
$\Rightarrow |Pq| + |qr| = |Pr|$

$\therefore$ The given points are collinear.

3) Show that the points $A(-3,-4)$, $B(0,2)$, $C(2,6)$ are collinear.

Sol:

$|AB| = \sqrt{(0-(-3))^2 + (2-(-4))^2}$



$= \sqrt{(0+3)^2 + (2+4)^2} = \sqrt{3^2 + 6^2} = \sqrt{9+36}$

$= \sqrt{45} = \sqrt{9\times 5} = 3\sqrt{5}$

$|BC| = \sqrt{(2-0)^2 + (6-2)^2} = \sqrt{2^2 + 4^2}$

$= \sqrt{4+16} = \sqrt{20} = \sqrt{4\times 5} = 2\sqrt{5}$

$|AC| = \sqrt{(-3-2)^2 + (-4-6)^2}$

$= \sqrt{(-5)^2 + (-10)^2} = \sqrt{25+100} = \sqrt{125} = \sqrt{5\times 25}$

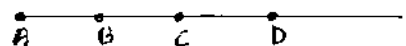
$= 5\sqrt{5}$

Now $|AB| + |BC| = 3\sqrt{5} + 2\sqrt{5} = (3+2)\sqrt{5} = 5\sqrt{5} = |AC|$

$\therefore |AB| + |BC| = |AC|$

Hence points A, B and C are collinear.

4. Show that the points $A(-9,-5)$, $B(-3,-2)$, $C(3,1)$ and $D(9,4)$ are collinear.



Sol: $|AB| = \sqrt{(-9-(-3))^2 + (-5-(-2))^2}$

$= \sqrt{(-9+3)^2 + (-5+2)^2} = \sqrt{(-6)^2 + (-3)^2} = \sqrt{36+9} = \sqrt{45}$

$= \sqrt{9\times 5} = 3\sqrt{5}$

$|BC| = \sqrt{(-3-3)^2 + (-2-1)^2} = \sqrt{(-6)^2 + (-3)^2} = \sqrt{36+9}$

$= \sqrt{45} = \sqrt{9\times 5} = 3\sqrt{5}$

$|CD| = \sqrt{(9-3)^2 + (4-1)^2} = \sqrt{6^2 + 3^2} = \sqrt{36+9} = \sqrt{45}$

$= \sqrt{9\times 5} = 3\sqrt{5}$

Now $|AB| + |BC| + |CD| = 3\sqrt{5} + 3\sqrt{5} + 3\sqrt{5} = (3+3+3)\sqrt{5}$

$= 9\sqrt{5}$ ——— (i)

$|AD| = \sqrt{(-9-9)^2 + (-5-4)^2}$

$= \sqrt{(-18)^2 + (-9)^2} = \sqrt{324+81} = \sqrt{405}$

$= \sqrt{81\times 5} = 9\sqrt{5}$ ——— (ii)

From (i) and (ii) we get that

$|AB| + |BC| + |CD| = |AD|$

$\therefore$ A, B, C and D are collinear points.

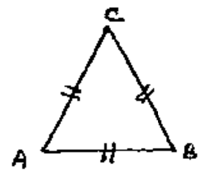
(283)

EXERCISE 10.3

1. Use distance formula to show that the points develop an equilateral triangle.

(i) (5,8), (5,12), $(5+2\sqrt{3}, 10)$

Sol: Let $A(5,8)$, $B(5,12)$, $C(5+2\sqrt{3}, 10)$ be the vertices of any triangle. then by distance formula:



$|AB| = \sqrt{(5-5)^2+(12-8)^2} = \sqrt{0+4^2}$
$= \sqrt{16} = 4$

$|BC| = \sqrt{(5-(5+2\sqrt{3}))^2+(12-10)^2}$
$= \sqrt{(5-5-2\sqrt{3})^2+(2)^2}$
$= \sqrt{(-2\sqrt{3})^2+4} = \sqrt{12+4} = \sqrt{16} = 4$

$(-2\sqrt{3})^2$
$=(-2)^2(\sqrt{3})^2$
$=4\times3=12$

$|AC| = \sqrt{(5-(5+2\sqrt{3}))^2+(8-12)^2}$
$= \sqrt{(5-5-2\sqrt{3})^2+(-2)^2} = \sqrt{12+4} = \sqrt{16} = 4$

As $|AB| = |BC| = |AC|$

Therefore A, B, C are the vertices of equilateral triangle

(ii) (0,8), (0,−8), $(8\sqrt{3}, 0)$

Sol: Let $A(0,8)$, $B(0,-8)$, $C(8\sqrt{3},0)$ be the vertices of any triangle then by distance formula:

$|AB| = \sqrt{(0-0)^2+(8-(-8))^2} = \sqrt{0+(16)^2} = \sqrt{256} = 16$ —(i)

$|BC| = \sqrt{(0-8\sqrt{3})^2+(-8-0)^2} = \sqrt{(-8\sqrt{3})^2+(-8)^2}$
$= \sqrt{64\times3+64} = \sqrt{192+64} = \sqrt{256} = 16$ —(ii)

$(-8\sqrt{3})^2$
$(-8)^2(\sqrt{3})^2$
64×3

$|AC| = \sqrt{(8\sqrt{3}-0)^2+(0-8)^2} = \sqrt{(8\sqrt{3})^2+64} = \sqrt{192+64}$
$= \sqrt{256} = 16$ ——(iii)

(284)

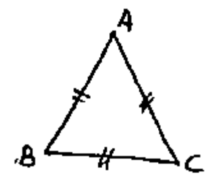
From Eq (i), (ii) and (iii) we get

$|AB| = |BC| = |AC|$

∴ The points A, B and C are the vertices of equilateral triangle

(iii) $(-6,0)$, $(10,0)$, $(2, 8\sqrt{3})$

Sol: Let $A(-6,0)$, $B(10,0)$, $C(2, 8\sqrt{3})$ be the vertices of any triangle. Then



$|AB| = \sqrt{(-6-10)^2 + (0-0)^2}$
$= \sqrt{(-16)^2 + 0} = \sqrt{256+0} = 16$

$|BC| = \sqrt{(10-2)^2 + (0-8\sqrt{3})^2}$
$= \sqrt{(8)^2 + (-8\sqrt{3})^2}$
$= \sqrt{64 + 64\times3} = \sqrt{64+192} = \sqrt{256} = 16$

$(-8\sqrt{3})^2$
$(-8)^2(\sqrt{3})^2$
$64\times3 = 192$

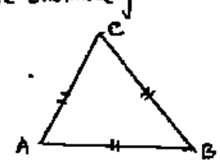
$|AC| = \sqrt{(-6-2)^2 + (0-8\sqrt{3})^2}$
$= \sqrt{(-8)^2 + (-8\sqrt{3})^2}$
$= \sqrt{64+192} = \sqrt{256} = 16$

As $|AB| = |BC| = |AC|$

⇒ The points A, B and C are the vertices of equi triangle.

(iv) $(-3,-2)$, $(-3,4)$, $(-3+3\sqrt{3}, 1)$

Sol: Let $A(-3,-2)$, $B(-3,4)$ and $C(-3+3\sqrt{3}, 1)$ be the vertices of any triangle. Then using the distance formula



$|AB| = \sqrt{(-3-(-3))^2 + (-2-4)^2}$
$= \sqrt{(-3+3)^2 + (-6)^2} = \sqrt{0+36} = 6$

$|BC| = \sqrt{(-3+3\sqrt{3}+3)^2 + (1-4)^2}$
$= \sqrt{(3\sqrt{3})^2 + (-3)^2}$
$= \sqrt{27+9} = \sqrt{36}$
$= 6.$

$(3\sqrt{3})^2$
$= (3)^2(\sqrt{3})^2$
$= 9\times3 = 27$

(285)

$|AC| = \sqrt{(-3+3\sqrt{3}-(-3))^2 + (-2-1)^2}$

$= \sqrt{(-3+3\sqrt{3}+3)^2 + (-3)^2}$

$= \sqrt{(3\sqrt{3})^2 + 9} = \sqrt{27+9} = \sqrt{36} = 6$

As $|AB| = |BC| = |AC|$, Therefore, A, B, C are the vertices of equilateral triangle.

2. Use distance formula to show that the following points are the vertices of an isosceles triangle.

i) $(-1,6)$, $(3,2)$, $(6,9)$

Note: A triangle in which two sides are equal is called isosceles triangle.

Sol: Let $A(-1,6)$, $B(3,2)$, $C(6,9)$ be the vertices of any triangle then,

$|AB| = \sqrt{(-1-3)^2 + (6-2)^2}$

$= \sqrt{(-4)^2 + (4)^2} = \sqrt{16+16} = \sqrt{32}$

$|BC| = \sqrt{(3-6)^2 + (2-9)^2}$

$= \sqrt{(-3)^2 + (-7)^2} = \sqrt{9+49} = \sqrt{58}$ ——(i)

$|AC| = \sqrt{(-1-6)^2 + (6-9)^2}$

$= \sqrt{(-7)^2 + (-3)^2} = \sqrt{49+9} = \sqrt{58}$ ——(ii)

From (i) and (ii) we have;

$|BC| = |AC| \Rightarrow$ A, B, C are the vertices of isosceles triangle.

(ii) $(4,7)$, $(5,0)$, $(-3,8)$

Sol: Let $A(4,7)$, $B(5,0)$ and $C(-3,8)$ be the vertices of any triangle. Then,

$|AB| = \sqrt{(5-4)^2 + (0-7)^2} = \sqrt{(1)^2 + (-7)^2}$

$= \sqrt{1+49} = \sqrt{50}$ ——(i)

(286)

$|BC| = \sqrt{(5-(-3))^2 + (0-8)^2}$

$= \sqrt{(5+3)^2 + (-8)^2}$

$= \sqrt{8^2 + (-8)^2} = \sqrt{64+64} = \sqrt{128}$ —— (ii)

$|AC| = \sqrt{(-3-4)^2 + (8-7)^2}$

$= \sqrt{(-7)^2 + (1)^2} = \sqrt{49+1} = \sqrt{50}$ —— (iii)

From (i), (ii) and (iii) we get

$|AB| = |AC|$.

Therefore, A, B and C are the vertices of isosceles triangle.

(iii) $(-4,6)$, $(4,-2)$, $C(5,7)$

Sol: Let $A(-4,6)$, $B(4,-2)$ and $C(5,7)$ be the vertices of any triangle. Then;

$|AB| = \sqrt{(-4-4)^2 + (6-(-2))^2}$

$= \sqrt{(-8)^2 + (6+2)^2} = \sqrt{(-8)^2 + (8)^2} = \sqrt{64+64}$

$= \sqrt{128}$ —— (i)

$|BC| = \sqrt{(4-5)^2 + (-2-7)^2}$

$= \sqrt{(-1)^2 + (-9)^2} = \sqrt{1+81} = \sqrt{82}$ —— (ii)

$|AC| = \sqrt{(-4-5)^2 + (6-7)^2}$

$= \sqrt{(-9)^2 + (-1)^2}$

$= \sqrt{81+1} = \sqrt{82}$ —— (iii)

From (i), (ii) and (iii) we get that $|BC| = |AC|$

Therefore, A, B and C are the vertices of isosceles triangle.

(iv) $(0,6)$, $(4,2)$, $C(7,9)$.

Sol: Let $A(0,6)$, $B(4,2)$, $C(7,9)$ be the vertices of any triangle

(287)

$|AB| = \sqrt{(4-0)^2+(2-6)^2}$

$= \sqrt{4^2+(-4)^2} = \sqrt{16+16} = \sqrt{32}$ ——(i)

$|BC| = \sqrt{(4-7)^2+(2-9)^2}$

$= \sqrt{(-3)^2+(-7)^2} = \sqrt{9+49} = \sqrt{58}$ ——(ii)

$|AC| = \sqrt{(0-7)^2+(6-9)^2}$

$= \sqrt{(-7)^2+(-3)^2} = \sqrt{49+9} = \sqrt{58}$ ——(iii)

From (i), (ii) and (iii) we get $|BC| = |AC|$

$\therefore$ A, B, C are the vertices of isosceles triangle.

3. Use distance formula to show that the following points represent a scalene triangle:

(i) (-2,5), (4,3), (7,-5)

Sol: Let A(-2,5), B(4,3), C(7,-5) be the vertices of any triangle then

> Note: A triangle in which all the side are different i.e $|AB| \neq |BC| \neq |AC|$ is called scalene triangle.

$|AB| = \sqrt{(-2-4)^2+(5-3)^2}$

$= \sqrt{(-6)^2+(2)^2} = \sqrt{36+4} = \sqrt{40}$ ——(i)

$|BC| = \sqrt{(7-4)^2+(-5-3)^2} = \sqrt{(3)^2+(-8)^2}$

$= \sqrt{9+64} = \sqrt{73}$ ——(ii)

$|AC| = \sqrt{(-2-7)^2+(5-(-5))^2}$

$= \sqrt{(-9)^2+(5+5)^2} = \sqrt{81+100} = \sqrt{181}$ ——(iii)

From (i), (ii) and (iii) we get that $|AB| \neq |BC| \neq |AC|$

Therefore, A, B and C are the vertices of a scalene triangle.

(288)

(ii) (2,1), (6,1), (5,5)

Sol: Let A(2,1), B(6,1), C(5,5) be the vertices of any triangle then; by using distance formula.

$|AB| = \sqrt{(6-2)^2+(1-1)^2} = \sqrt{4^2+0^2} = \sqrt{16} = 4$ ——(i)

$|BC| = \sqrt{(5-6)^2+(5-1)^2}$

$= \sqrt{(-1)^2+(4)^2} = \sqrt{1+16} = \sqrt{17}$ ——(ii)

$|AC| = \sqrt{(5-2)^2+(5-1)^2} = \sqrt{(3)^2+(4)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$ ——(iii)

From (i), (ii) and (iii) we have $|AB| \neq |BC| \neq |AC|$

∴ A, B and C are the vertices of scalene triangle.

(iii) (−5,−4), (2,3), (−2,4).

Sol: Let A(−5,−4), B(2,3), C(−2,4) be the vertices of any triangle. then by distance formula.

$|BC| = \sqrt{(2-(-2))^2+(3-4)^2}$

$= \sqrt{(2+2)^2+(-1)^2} = \sqrt{4^2+1^2} = \sqrt{16+1} = \sqrt{17}$ ——(i)

$|AB| = \sqrt{(-5-2)^2+(-4-3)^2}$

$= \sqrt{(-7)^2+(-7)^2} = \sqrt{49+49} = \sqrt{98}$ ——(ii)

$|AC| = \sqrt{(-5-(-2))^2+(-4-4)^2}$

$= \sqrt{(-5+2)^2+(-8)^2} = \sqrt{(-3)^2+(-8)^2}$

$= \sqrt{9+64}$ ——(iii)

From (i), (ii) and (iii) we get $|AB| \neq |BC| \neq |AC|$

So, A, B and C are the vertices of Scalene triangle.

(iv) $(-10,4)$, $(7,1)$, $(-3,-3)$

Sol: Let $A(-10,4)$, $B(7,1)$, $C(-3,-3)$ be the vertices of any tringle then we have from distance formula.

$|AB| = \sqrt{(-10-7)^2+(4-1)^2} = \sqrt{(-17)^2+3^2}$

$= \sqrt{289+9} = \sqrt{298}$ ——— (i)

$|BC| = \sqrt{(7-(-3))^2+(1-(-3))^2}$

$= \sqrt{(7+3)^2+(1+3)^2} = \sqrt{10^2+4^2} = \sqrt{100+16}$

$= \sqrt{116}$ ——— (ii)

$|AC| = \sqrt{(-10-(-3))^2+(4-(-3))^2}$

$= \sqrt{(-10+3)^2+(4+3)^2} = \sqrt{(-7)^2+(7)^2}$

$= \sqrt{49+49} = \sqrt{98}$ ——— (iii)

From Eq (i), (ii), (iii) $|AB| \neq |BC| \neq |AC|$

So it a scalene triangle.

4. Use distance formula, to show that the following points form a right angle triangle

(i) $(-2,3)$, $(4,3)$, $(-2,11)$

Let $A(-2,3)$, $B(4,3)$, $C(-2,11)$ be the vertices of any triangle. Then by distance formula:

$|AB| = \sqrt{(-2-4)^2+(3-3)^2}$

$= \sqrt{(-6)^2+(0)^2} = \sqrt{36+0} = \sqrt{36} = 6$

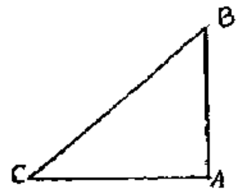
$|BC| = \sqrt{(4-(-2))^2+(3-11)^2} = \sqrt{(4+2)^2+(-8)^2}$

$= \sqrt{(6)^2+8^2} = \sqrt{36+64} = \sqrt{100} = 10$

(290)

$|AC| = \sqrt{(-2-(-2))^2+(11-3)^2}$

$= \sqrt{(-2+2)^2+8^2} = \sqrt{0+64} = 8$



Now, As $|AC|^2+|AB|^2=|BC|^2$ i.e

$8^2+6^2=10^2 \Rightarrow 100=100$

So ABC is right angled triangle.

(ii) $(1,4)$, $(1,7)$, $(8,7)$

Sol: Let $A(1,4)$, $B(1,7)$ and $C(8,7)$ be the vertices of any triangle then by distance formula:

$|AB| = \sqrt{(1-1)^2+(7-4)^2} = \sqrt{0^2+3^2} = \sqrt{9} = 3$

$|BC| = \sqrt{(8-1)^2+(7-7)^2} = \sqrt{7^2+0^2} = \sqrt{49} = 7$

$|AC| = \sqrt{(8-1)^2+(7-4)^2} = \sqrt{7^2+3^2} = \sqrt{49+9} = \sqrt{58}$

As $|AB|^2+|BC|^2=|AC|^2$ i.e

$(3)^2+(7)^2=(\sqrt{58})^2$

$9+49=58 \Rightarrow 58=58$

So the vertices A, B, C of triangle obeys Pythagorus Theorem so A, B, C are the vertices of right triangle

(iii) $(-6,5)$, $(-6,-7)$, $(-1,-7)$

Sol: Let $A(-6,5)$, $B(-6,-7)$, $C(-1,-7)$ be the vertices of any triangle, Then by distance formula:

$|AB| = \sqrt{(-6-(-6))^2+(5-(-7))^2}$

$= \sqrt{(-6+6)^2+(5+7)^2} = \sqrt{0+12^2} = \sqrt{144} = 12$

$|BC| = \sqrt{(-6-(-1))^2+(-7-(-7))^2}$

$|BC| = \sqrt{(-6+1)^2 + 0^2} = \sqrt{(-5)^2} = \sqrt{25} = 5$

$|AC| = \sqrt{(-6-(-1))^2 + (5-(-7))^2}$

$= \sqrt{(-6+1)^2 + (5+7)^2} = \sqrt{(-5)^2 + (12)^2}$

$= \sqrt{25+144} = \sqrt{169} = 13$

As $|AB|^2 + |BC|^2 = |AC|^2$ i.e

$(12)^2 + (5)^2 = (13)^2 \Rightarrow 169 = 169$

So by Pythagorus Theorem, A, B, C are the vertices of right triangle.

(iv) (1,4), (3,2), (6,9)

Sol: Let A(1,4), B(3,2) and C(6,9) be the vertices of any triangle then, by using distance formula.

$|AB| = \sqrt{(3-1)^2 + (2-4)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$

$|BC| = \sqrt{(3-6)^2 + (2-9)^2} = \sqrt{(-3)^2 + (-7)^2}$

$= \sqrt{9+49} = \sqrt{58}$

$|AC| = \sqrt{(6-1)^2 + (9-4)^2} = \sqrt{(5)^2 + (5)^2} = \sqrt{25+25} = \sqrt{50}$

As $|AB|^2 + |AC|^2 = |BC|^2$ i.e

$(\sqrt{8})^2 + (\sqrt{50})^2 = (\sqrt{58})^2 \Rightarrow 8 + 50 = 58$

$\Rightarrow 58 = 58$

$\therefore$ By Pythagorus Theorem A, B and C are the vertices of right angled triangle

REVISION EXERCISE-10

1. Which of the following statements are true and which are false.

(i) Coordinate Geometery is also called analytic Geometery. (T)

(ii) Coordinate plane represents a two dimentional plane. (T)

(iii) On X-axis, points to the right of orign represents positive real numbers (T)

(iv) Point (4,-3) lies in quadrant(III) (F)

(v) Distance between (6,0) and (0,8) is 10 (T)

(vi) Three collinear points form a triangle (F)

(vii) In a triangle ABC, if |AB| = |BC| then the triangle is an equilateral one. (F)

2. Fill in the blanks:

(i) (-6,-8) lies in quadrant 3rd

(ii) On Y-axis, the points below the orign are associated with Negetive real numbers

(iii) In quadrant II, X-coordinate is Negetive

(iv) Point (0,-4) lies on Y-axis

3. Choose the correct option from the given.

(i) a (ii) b (iii) d (iv) d (v) d

END

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